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Prize-Linked Savings Games: Theory and Experiment[1]

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September 4, 2018

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1 Introduction

A low savings rate has long been a concern among economists. Recent changes in socioeconomic factors, such as fluid labor markets, modest long-run financial returns, and hyper-longevity, call for new retirement-savings schemes for today's younger workers. Policy makers have attempted to employ schemes to encourage savings, such as improving financial literacy (Choi et al., 2011; Lusardi & Mitchell, 2014), providing tax incentives (Duflo et al., 2006, 2007), and nudging with default options (Beshears et al., 2009; Carroll et al., 2009). One lesser-known instrument is an alternative savings product that bundles savings and a lottery, so-called prize-linked savings (PLS). A PLS saver has a chance to win a prize without losing the principal; a greater PLS amount provides a greater chance of winning a prize. In other words, PLS savers virtually receive a lottery ticket for every deposit of a certain amount. Due to the possibility of earning a larger return from savings, PLS products conceivably appeal to consumers, in particular those who regularly buy lottery tickets.

PLS products are available in many countries. The best-known PLS product is the British Premium Bonds program, offered by National Savings and Investments since 1956. As of December 2017, there were 21 million Premium Bond holders who could win one of the two monthly jackpots, one million GBP each, and the total outstanding balance of the Premium Bonds was 72 billion GBP (approx. 100 billion USD).[2] PLS is also a popular savings option in Thailand. At the end of 2017, the Government Savings Bank, which has run PLS programs since 1943, had more than 900 billion THB (approx. 29 billion USD) outstanding PLS deposits. Its three-year PLS program draws three 10-million THB jackpots and other smaller prizes every month.[3] PLS products are also available in many countries such as Japan, Germany, Spain, Mexico, and Argentina (see a survey by Guillen & Tschoegl, 2002). While PLS is not common in the United States,[4] Tufano et al. (2011) suggest it might appeal to U.S. consumers.

[2] National Savings and Investments' website (https://www.nsandi.com/premium-bonds).
[3] Government Savings Bank's website (http://www.gsb.or.th).
[4] As of June 2017, 20 states, including Arizona, Arkansas, Connecticut, Illinois, Indiana, Kansas, Louisiana, Maine, Maryland, Michigan, Minnesota, Nebraska, New Jersey, New York, North Carolina, Oregon, Rhode Island, South Carolina, Virginia, and Washington, allow credit unions and other financial institutions to hold savings promotion raffles (http://www.ncsl.org/research/financial-services-and-commerce/prize-linked-savings-2016-legislation.aspx).

There are many forms of PLS. Some PLS accounts pay a below-average fixed interest rate in addition to a chance of winning big prizes.[5] Some do not return all of the principal, so the ex post rate of return for nonwinners is negative.[6] Another variation of PLS is a marketing campaign that provides a chance of winning a prize to new customers or to existing customers who increase their balance.[7]

While PLS products are intriguing alternatives for some financial institutions to raise funds, an interesting flip side is whether and how PLS products can encourage household savings. Hence, we theoretically and experimentally address these questions. In most PLS products, the probability that an account holder will win a prize depends on her deposit amount as well as those of other account holders. This strategic interaction in PLS is similar to that in Tullock's contest (1980), which is the basis of our PLS game. Specifically, we consider a principal-preserving PLS that pays a single prize to a randomly selected PLS winner. Each player allocates her endowment among three options: an early payment, a traditional savings (TS) account that pays a fixed interest rate, and a PLS account.

By assuming that all players are expected utility maximizers with concave constant absolute risk aversion (CARA) utility functions, we prove that there exists a unique Nash equilibrium even with heterogeneous degrees of risk aversion. We also derive each player's optimal choices of early payment, TS deposit, and PLS deposit as functions of the number of players, their preference parameters, the TS interest rate, and the PLS prize. We compare total savings in scenarios with and without PLS accounts. According to our theory, under the assumption of homogeneous players, every player, including those who do not save when a TS account is the only savings option, will save in a PLS account when available. However, PLS will cannibalize TS and reduce total savings if the initial TS deposit is high.

Our experiment to test these theoretical predictions is the first to analyze PLS with strategic interactions. Each subject in a random group of five allocates her endowment in 34 scenarios

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[5] The only two PLS product providers in Thailand, the Government Savings Bank and the Bank for Agriculture and Agricultural Cooperatives, offer a fixed interest rate that varies with the market rate.
[6] This variation of PLS is called lotto-linked savings (LLS). Dizon & Lybbert (2017) conducted a lab-in-field experiment to evaluate LLS in Haiti.
[7] For example, the Emirates NBD Bank in the United Arab Emirates offered a chance to win an Aston Martin DB9 (approx. 230,000 USD) or Omega Speedmaster watch (approx. 4,600 USD) to new customers or to existing customers who increased their balance. See https://www.emiratesnbd.com/en/campaign-form/aston-martin/ for details.

with one or two savings options (i.e., TS account only, PLS account only, and both TS and PLS accounts) and different parameters (i.e. TS interest rate and PLS prize).

Our experimental results suggest that PLS would be an effective way to encourage savings. When subjects are allowed to save in a PLS account in addition to a TS account, most of them choose to save in both. Most importantly, their total savings increase, although the two accounts are possibly substitutable. Furthermore, PLS encourages almost half of those who do not save to start saving. Lastly, the effectiveness of PLS varies across subjects depending on their preference regarding savings options. While saving at the lowest rate, subjects who prefer PLS display the greatest increase in total savings and the highest ratio of PLS deposit to total savings.

These findings are consistent with previous studies, despite strategic interaction in our design.[8] Atalay et al. (2014), the closest study to ours, analyze subjects' allocation decisions among four potential alternatives: immediate payment, immediate lottery, TS, and PLS. They observe an increase in total savings and a decrease in lottery expenditures after introducing PLS. Even though Atalay et al. (2014) and our studies use different settings,[9] the increases in total savings due to PLS are comparable (around 12 percentage points). In contrast, the subjects in Filiz-Ozbay et al. (2015) make binary decisions between an immediate payment and savings in either a TS or a PLS account. They defer payment at a greater rate with PLS than with TS of the same expected return. Furthermore, the results are stronger among male and self-reported lottery-ticket buyers.

Dizon and Lybbert (2017) adopt Atalay et al. (2014)'s setting, but replace PLS with lotto-linked savings (LLS) in a lab-in-field experiment in Port-au-Prince, Haiti. Their LLS account may not preserve the entire principal but guarantees a minimum return ranging from 60%–100%.[10] They find that subjects save more when LLS is available, and the savings rate is higher when LLS returns more principal with less lotto gambling. They also find that LLS is

[8] The only exception is the study by Loibl et al. (2018), who find a PLS product has no additional impact on savings.
[9] Our design is different from that in Atalay et al. (2014) in four important ways. First, the probability of winning the PLS prize depends on all players rather than being predetermined. Second, the interest rates are closer to market interest rates and the savings period is longer (180 days). Third, each subject receives compensation from one random scenario. Last, there is no lottery purchase option.
[10] PLS is a special case of LLS with a 100% minimum return.

more effective among subjects who overweight small probabilities and those who spend more on lotto gambling.[11]

The benefit of PLS is also consistent with the empirical study by Cole et al. (2018) on the impact of the PLS accounts offered by a commercial bank in South Africa. They find two important results. First, PLS has a stronger positive effect on those who do not have TS accounts than on those who do. Second, PLS does not cannibalize TS. In fact, they find that branches with a higher amount of PLS deposits observe a greater increase in TS deposits, and individuals with PLS accounts typically increase their balances in TS accounts. In our experiment, we find that PLS and TS could be substitutes or complements.

Based on a field experiment conducted in Mexico, Gertler et al. (2018) study the effectiveness of a bank's promotional campaign similar to PLS. In the treatment branches, clients who opened a new account or increased a savings balance received a chance to win a cash prize. The researchers show that the number of accounts opened in these branches is higher than the number opened in the control branches. Even after the end of the campaign, clients in the treatment group have kept their new accounts and continue to save at the same rates as those in the control group.

Our main contributions to the literature are as follows. First, we develop a game-theoretic model of allocation decisions involving PLS. Unlike the existing literature, which assumes a predetermined probability of winning a PLS prize, we incorporate strategic interactions among PLS account holders into the theory and the experiment. Our approach is more suitable for analyzing a PLS scheme in which an account holder's probability of winning a prize depends on her deposit amount as well as those of other account holders. It allows us to assess the effect of PLS on a smaller scale, where the number of players and each player's actions are more relevant. Moreover, this study contributes to the literature of rent-seeking contests (see Corchón, 2007; Konrad, 2009; Dechenaux et al., 2015, for surveys), since our PLS game is a Tullock contest variation in which each player fully recovers her investment in a future period regardless of the contest results. Second, our experimental design of portfolio allocation can show a pattern of substitutability and complementarity between TS and PLS.

[11] This result is consistent with the study by Cookson (2018), who also finds a substitution-effect between PLS and gambling in Nebraska. He shows that individuals who have access to PLS reduce gambling by at least 3% more than those who do not.

This complements the empirical study, which cannot observe the entire portfolio and movement across assets.[12]

The outline of this paper is the following. In the next section, we describe a theoretical model. Section 3 lays out the experimental design. Section 4 presents results from the experiments. Section 5 concludes our findings.

2 Theoretical Model

We analyze a two-period model with n players. For $i = 1, \dots, n$, player i is given income I_{it} and derives utility from consumption in period t, denoted by $c_{it} \geq 0$. We assume that player i's utility function in each period is given by

$$u_i(c_{it}) = -e^{-\alpha_i c_{it}}$$

where $\alpha_i > 0$. This utility function exhibits strict concavity in c_{it} and constant absolute risk aversion (CARA). In period 1, each player chooses to save $x_i \geq 0$ in a traditional savings (TS) account and $y_i \geq 0$ in a prize-linked savings (PLS) account, such that $x_i + y_i \leq I_{i1}$. The TS account pays interest in period 2 at rate $r > 0$. The PLS account does not pay interest, but one of the account holders will be randomly chosen as the prizewinner and paid $R > 0$ in period 2. Regardless of whether player i wins the prize, she will receive y_i back in period 2. Therefore, consumption in the first period is $c_{i1} = I_{i1} - x_i - y_i$, followed by $c_{i2} = I_{i2} + x_i(1 + r) + y_i + R$ if she wins the prize, and $c_{i2} = I_{i2} + x_i(1 + r) + y_i$ if she does not win. The probability that player i wins the prize is $y_i/(y_i + Y_{-i})$, where $Y_{-i} = \sum_{j \neq i}^{n} y_j$.[13] We let β_i be player i's discount factor for her utility in period 2 so that her expected utility over the two periods can be written as

$$U_i(x_i, y_i) = u_i(I_{i1} - x_i - y_i) + \beta_i \frac{y_i}{y_i + Y_{-i}} u_i(I_{i2} + x_i(1 + r) + y_i + R) + \beta_i \frac{Y_{-i}}{y_i + Y_{-i}} u_i(I_{i2} + x_i(1 + r) + y_i)$$

[12] As Cole et al. (2018) concede, their empirical finding cannot rule out the possibility that PLS holders may shift some of their savings from other banks to the bank that offered the PLS accounts, for their convenience.
[13] If $y_i = 0$ for all i, the probability is not defined, so we impose that there is no PLS winner.

2.1 Best Responses and Equilibrium

We define $\Delta I_i = I_{i1} - I_{i2}$. Setting $\frac{\partial U_i}{\partial x_i} = 0$ and $\frac{\partial U_i}{\partial y_i} = 0$ yields the first-order conditions,

$$\frac{\beta_i(1+r)}{y_i + Y_{-i}}(y_i e^{-\alpha_i R} + Y_{-i}) = e^{-\alpha_i(\Delta I_i - (2+r)x_i - 2y_i)} \tag{1}$$

and

$$\frac{\beta_i}{y_i + Y_{-i}}\left(\frac{Y_{-i}}{y_i + Y_{-i}} \cdot \frac{1 - e^{-\alpha_i R}}{\alpha_i} + y_i e^{-\alpha_i R} + Y_{-i}\right) = e^{-\alpha_i(\Delta I_i - (2+r)x_i - 2y_i)}, \tag{2}$$

respectively. Let x_i^* and y_i^* be the choice variables that maximize $U_i(x_i, y_i)$.[14] Since the terms on the right-hand side of Equations (1) and (2) are identical, we can set the terms on the left-hand side of both equations as equal, i.e.,

$$\frac{\beta_i(1+r)}{y_i + Y_{-i}}(y_i^* e^{-\alpha_i R} + Y_{-i}) = \frac{\beta_i}{y_i^* + Y_{-i}}\left(\frac{Y_{-i}}{y_i^* + Y_{-i}} \cdot \frac{1 - e^{-\alpha_i R}}{\alpha_i} + y_i^* e^{-\alpha_i R} + Y_{-i}\right)$$

which implies the following quadratic equation,

$$e^{-\alpha_i R} y_i^{*2} + (1 + e^{-\alpha_i R})Y_{-i} y_i^* + \left[Y_{-i}^2 - Y_{-i}\left(\frac{1 - e^{-\alpha_i R}}{\alpha_i r}\right)\right] = 0.$$

Therefore, player i's best response function is the positive solution of the above equation, i.e.,

$$y_i^* = \frac{Y_{-i}}{2e^{-\alpha_i R}}\left[-(1 + e^{-\alpha_i R}) + (1 - e^{-\alpha_i R})\sqrt{1 + \frac{4e^{-\alpha_i R}}{\alpha_i r(1 - e^{-\alpha_i R})Y_{-i}}}\right]. \tag{3}$$

Such a solution does not exist when the bracketed sum on the right-hand side of Equation (3) is negative, or equivalently, when $Y_{-i} \geq \kappa_i$ where

$$\kappa_i = \frac{1 - e^{-\alpha_i R}}{\alpha_i r}.$$

We call κ_i the PLS deposit threshold for player i. If the sum of the other players' PLS deposits is greater than or equal to κ_i, player i's best response is to choose $y_i^* = 0$.

Example 1. Let $n = 2$, $r = 0.1$, and $R = 1$. If $\alpha_1 = \alpha_2 = 1$, then $\kappa_1 = \kappa_2 = 6.321$ and

---

[14] Here we only consider interior solutions so that the optimal choices of x_i^* and y_i^* satisfy both Equations (1) and (2). If we let $r = 0$ while $R > 0$, then $x_i^* = 0$ and we can solve for y_i^* from Equation (2). On the other hand, if we let $R = 0$ while $r > 0$, then $y_i^* = 0$ and we can solve for x_i^* from Equation (1).

$$y_i^* = \begin{cases} \frac{y_j}{2e^{-1}}\left[-(1+e^{-1}) + (1-e^{-1})\sqrt{1+\frac{4e^{-1}}{0.1(1-e^{-1})y_j}}\right] & ;\text{if } y_j < 6.321 \\ 0 & ;\text{if } y_j \geq 6.321 \end{cases}$$

for $i = 1, 2$ and $i \neq j$. See each player's best-response function in Figure 1. The two best-response curves cross only once, at $y_1 = y_2 = 2.310$, which is the unique Nash equilibrium of the game (see point A). Both players have an equal chance (50%) of winning the PLS prize.

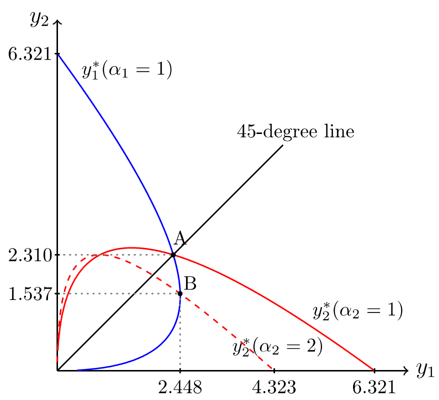

Figure 1. Best-response functions and equilibria in Examples 1 and 2

Example 2. Let $n = 2$, $r = 0.1$, and $R = 1$. Let $\alpha_1 = 1$ so player 1's best-response function is the same as y_1^* in Example 1. If $\alpha_2 = 2$, then $\kappa_2 = 4.323$. It follows that player 2's best response becomes

$$y_2^* = \begin{cases} \frac{y_1}{2e^{-2}}\left[-(1+e^{-2}) + (1-e^{-2})\sqrt{1+\frac{4e^{-1}}{0.2(1-e^{-2})y_1}}\right] & ;\text{if } y_1 < 4.323 \\ 0 & ;\text{if } y_1 \geq 4.323 \end{cases}$$

which is represented by the dashed curve in Figure 1. Given $\alpha_1 = 1$ and $\alpha_2 = 2$, player 2 is more risk-averse than player 1, and as a result, player 1 allocates more money in a PLS account than player 2 does. We observe that the two best-response curves cross at $y_1 = 2.448$ and $y_2 = 1.537$, which is the unique Nash equilibrium of the game (see point B). The total PLS by the two players is 3.985, and player 1 has a larger chance of winning the PLS prize than does player 2 (61.4% versus 38.6%).

2.2 Equilibrium with Multiple Players

The above examples demonstrate how we derive the equilibrium in a PLS game with $n = 2$. Next, we formally show that the game has a unique equilibrium given $n \geq 2$ heterogeneous players. We follow Cornes and Hartley's (2003) share function approach to identify an equilibrium.[15] Specifically, we define player i's share $s_i(\psi) = \frac{y_i^*}{\psi}$, where $\psi = y_i^* + Y_{-i} > 0$,[16] to be her share of the total PLS deposits by all players, including herself. This is also equal to her probability of winning the PLS prize, which corresponds to her best response to Y_{-i}. Equation (3) implies that player i's optimal PLS amount is y_i^* such that

$$r(y_i^* e^{-\alpha_i R} + Y_{-i}) = \frac{Y_{-i}}{y_i^* + Y_{-i}} \cdot \frac{1 - e^{-\alpha_i R}}{\alpha_i}.$$

Given the definition of $s_i(\psi)$, the above condition can be written as

$$r[s_i \psi e^{-\alpha_i R} + (1 - s_i)\psi] = (1 - s_i)\frac{1 - e^{-\alpha_i R}}{\alpha_i}$$

which is equivalent to

$$\alpha_i r \psi (s_i e^{-\alpha_i R} - s_i) + s_i(1 - e^{-\alpha_i R}) = 1 - e^{-\alpha_i R} - \alpha_i r \psi.$$

Therefore,

$$s_i = \frac{1 - e^{-\alpha_i R} - \alpha_i r \psi}{1 - e^{-\alpha_i R} - (1 - e^{-\alpha_i R})\alpha_i r \psi} = \frac{\kappa_i - \psi}{\kappa_i - (1 - e^{-\alpha_i R})\psi} \quad (4)$$

given $\psi \in (0, \kappa_i)$, and 0 given $\psi \geq \kappa_i$. Note that this probability is written as a function of total PLS deposit by all participants. It follows that we can derive the total PLS deposit in equilibrium by setting the sum of all shares, i.e., $\sum_{i=1}^{n} s_i(\psi)$, as equal to one. We call such a value ψ^e and find that, in equilibrium, player i saves $s_i \psi^e$ in a PLS account. The following proposition shows that ψ^e is unique and that every PLS game has a unique equilibrium.

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[15] The share functions approach has been used to identify the equilibrium given heterogeneous risk-averse or risk-loving players in various aggregative games (see Cornes & Hartley, 2005; Jindapon & Whaley, 2015; Jindapon & Yang, 2017).

[16] Note that ψ must be strictly positive, since player i's best response to $Y_{-i} = 0$ is to save a very small positive amount in a PLS account.

Proposition 1. *Given n heterogeneous CARA players, a PLS game has a unique Nash equilibrium.*

Proof. See Appendix.

Example 1 Revisited. Given $n = 2$ and $\alpha_1 = \alpha_2 = 1$, the share functions are identical, as illustrated in Figure 2(a). There is only one value of ψ such that $s_1 + s_2 = 1$. That value is 4.620, and since $s_1(4.620) = s_2(4.620) = 0.5$, it follows that $y_1^e = y_2^e = 0.5 \times 4.620 = 2.310$.

Example 2 Revisited. Given $n = 2$, $\alpha_1 = 1$, and $\alpha_2 = 2$. We can plot each player's share function as in Figure 2(b). Each function strictly decreases for $\psi \in (0, \kappa_i)$, and there is only one value of ψ such that $s_1 + s_2 = 1$. That value is 3.985, which is the sum of PLS deposits by both players in equilibrium. Since $s_1(3.985) = 0.614$, we know that $y_1^e = 0.614 \times 3.985 = 2.448$. Similarly, $s_2(3.985) = 0.386$ implies that $y_2^e = 1.537$.

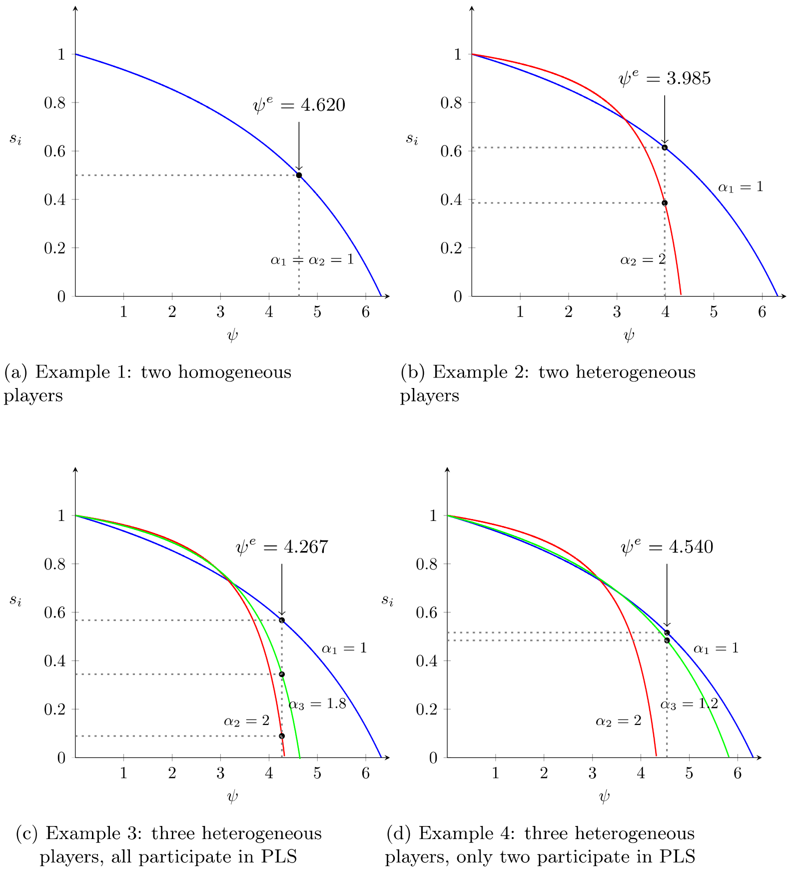
Example 3. Let $n = 3$, $\alpha_1 = 1$, $\alpha_2 = 2$, and $\alpha_3 = 1.8$. We can plot each player's share function as in Figure 2(c). Each function strictly decreases for $\psi \in (0, \kappa_i)$, and we find that 4.267 is only one value of ψ such that $s_1 + s_2 + s_3 = 1$. Since $s_1(4.267) = 0.567$, $s_2(4.267) = 0.089$, and $s_3(4.267) = 0.344$, we find that $y_1^e = 2.419$, $y_2^e = 0.379$, and $y_3^e = 1.469$, respectively.

Example 4. Let $n = 3$, $\alpha_1 = 1$, $\alpha_2 = 2$, and $\alpha_3 = 1.2$. We can plot each player's share function as in Figure 2(d).. All functions strictly decrease for $\psi \in (0, \kappa_i)$, and we find that 4.540 is the only value of ψ such that $s_1 + s_2 + s_3 = 1$. Since $\kappa_2 = 4.323 < 4.540$, player 2 will make no PLS deposit, i.e., $y_2^e = 0$. Since $s_1(4.540) = 0.516$ and $s_3(4.540) = 0.484$, we find that $y_1^e = 2.343$ and $y_3^e = 2.198$.

Given a player's PLS deposit in equilibrium, we can derive her corresponding TS deposit from the first-order conditions. Let x_i^e denote player i's an optimal TS deposit in equilibrium. Substituting y_i^e for y_i in Equation (1) yields

$$x_i^e = \frac{\ln(\beta_i(1+r)Z_i) + \alpha_i(\Delta I_i - 2y_i^e)}{\alpha_i(2+r)} \tag{5}$$

where $Z_i = \frac{\psi^e - y_i^e(1-e^{-\alpha_i R})}{\psi^e}$.



(a) Example 1: two homogeneous players

(b) Example 2: two heterogeneous players

(c) Example 3: three heterogeneous players, all participate in PLS

(d) Example 4: three heterogeneous players, only two participate in PLS

Figure 2. Share functions and equilibria in Examples 1 to 4

2.3 Symmetric Equilibrium and Comparative Statics

In this section, we consider a special case where players have homogeneous risk preferences, so that we can obtain an analytical solution and perform comparative statics analysis on each player's savings in equilibrium. If there are n players with $\alpha_i = \alpha$ for all i, we can derive a symmetric equilibrium by substituting $Y_{-i} = (n-1)y_i$ in Equation (3) or setting $s_i = 1/n$ in Equation (4). We can derive each player's PLS deposit in a symmetric equilibrium as

$$y_i^e = \frac{(n-1)(1-e^{-\alpha R})}{\alpha n r(n-1+e^{-\alpha R})}. \tag{6}$$

We find the following properties for y_i^e when $\alpha_i = \alpha$ for all i.

Proposition 2. *Properties of y_i^e with $\alpha_i = \alpha$ for all i:*

1) $\frac{\partial y_i^e}{\partial \alpha} < 0$
2) $\frac{\partial y_i^e}{\partial r} < 0$
3) $\frac{\partial y_i^e}{\partial R} > 0$
4) $\frac{\partial y_i^e}{\partial n} < 0$ *and* $\lim_{n\to\infty} y_i^e = 0$

Proof. See Appendix.

Proposition 2 suggests that in a symmetric equilibrium, players increase their PLS deposits as they become less risk-averse, the TS interest rate decreases, the PLS prize increases, or the number of players decreases.

If we further assume that $\beta_i = \beta$ and $\Delta I_i = \Delta I$ for all i, an optimal TS deposit x_i^e in Equation (5) and y_i^e in a symmetric equilibrium from Equation (6), we can derive the corresponding TS deposit given homogeneous players as

$$x_i^e = \frac{\ln\left(\frac{\beta(1+r)(n-1+e^{-\alpha R})}{n}\right) + \alpha(\Delta I - 2y_i^e)}{\alpha(2+r)}. \tag{7}$$

It follows that $x_i^e > 0$ if and only if

$$\beta(1+r) > \frac{ne^{-\alpha(\Delta I - 2y_i^e)}}{n-1+e^{-\alpha R}}. \tag{8}$$

2.4 Effects of Introducing PLS

We let a TS account be each player's only choice of saving in a status quo. First, we calculate each player's optimal TS deposit in the status quo. Then, we compare it to her savings allocation when a PLS deposit is available in addition to the TS account. We can categorize players in the status quo into two groups: (1) TS non-savers, i.e., those who do not save in a TS account, and (2) TS savers, i.e., those who do.

2.4.1 TS Non-Savers

In the status quo where PLS is unavailable, i.e., $R = 0$, we derive each player's optimal TS deposit from Equation (1) as

$$x_i^o = \frac{\ln\big(\beta(1+r)\big) + \alpha\Delta I}{\alpha(2+r)}. \tag{9}$$

It follows that $x_i^o > 0$ if and only if

$$\beta(1+r) > e^{-\alpha\Delta I}. \tag{10}$$

Thus, player i is a TS non-saver if $\beta(1+r) \leq e^{-\alpha\Delta I}$. If we allow PLS as an additional saving option, then all players have the same share function. It follows that each player will choose a positive PLS deposit in equilibrium. Since the right-hand side of Equation (8) is larger than that of Equation (10) for any $n \geq 2$, we find that a TS non-saver still makes no TS deposit when PLS becomes available. Due to her positive PLS deposit, we can conclude that PLS induces TS non-savers to save.

Remark 1 (TS Non-Savers). *Assume that all players are identical, i.e., $\alpha_i = \alpha$, $\beta_i = \beta$, and $\Delta I_i = \Delta I$ for all i. If $\beta(1+r) \leq e^{-\alpha\Delta I}$, all players are TS non-savers. Introducing PLS will induce them to save only in a PLS account.*

Intuitively, a player should save in a PLS account if no other player does, since participating guarantees the prize. Therefore, every player saves in a PLS account in a symmetric equilibrium. However, this result may not hold when players are heterogeneous. Some TS non-savers may still refrain from saving even when a PLS account is available because they are too risk-averse—see player 2 in Example 4.

2.4.2 TS Savers

Now we assume that $\beta(1+r) > e^{-\alpha\Delta I}$ so that $x_i^o > 0$ in the status quo. When PLS becomes available, like TS non-savers, all TS savers will save in a PLS account. We categorize TS savers into two groups: (1) Type-1 TS savers—those who no longer save in a TS account after introducing PLS, and (2) Type-2 TS savers—those who do. According to Equation (8), we know that if $\beta(1+r) < \frac{ne^{-\alpha(\Delta I - 2y_i^e)}}{n-1+e^{-\alpha R}}$, then we will obtain a corner solution with no TS deposit. That is, all players are Type-I savers. Otherwise, they are Type-2 TS savers.

To examine the effects of introducing PLS, we define $\Delta x_i = x_i^e - x_i^o$ as a change in TS deposits and $\Delta s_i = x_i^e + y_i^e - x_i^o$ as a change in total deposits. For Type-1 TS savers, we can solve for y_i^e from Equation (2) by letting $r = 0$ while $R > 0$. Since $x_i^* = 0$, it follows immediately that $\Delta x_i = -x_i^o < 0$ and that PLS is a substitute for TS.

Remark 2 (Type-1 TS Savers). *Assume that all players are identical, i.e.,* $\alpha_i = \alpha$, $\beta_i = \beta$, *and* $\Delta I_i = \Delta I$ *for all* i. *If* $e^{-\alpha\Delta I} < \beta(1+r) \leq \frac{ne^{-\alpha(\Delta I - 2y_i^e)}}{n-1+e^{-\alpha R}}$, *all players are Type-1 TS savers. Introducing PLS will decrease a player's TS deposit to zero, but its effect on total savings is ambiguous.*

We obtain the case of Type-1 TS savers only when $\beta(1+r)$ lies within the interval $\left(e^{-\alpha\Delta I}, \frac{ne^{-\alpha(\Delta I - 2y_i^e)}}{n-1+e^{-\alpha R}}\right)$. If $\beta(1+r)$ is close to the lower bound, a TS deposit under the status quo is relatively small, so introducing PLS increases total savings. On the other hand, if $\beta(1+r)$ is close to the upper bound, introducing PLS decreases total savings.

For Type-2 TS savers, we use interior solutions as derived in Section 2.3. A change in Player i's TS deposit when a PLS account is available can be written as

$$\Delta x_i = x_i^e - x_i^o = \frac{1}{\alpha(2+r)}\left[\ln\left(\frac{n-1+e^{-\alpha R}}{n}\right) - \frac{2(n-1)(1-e^{-\alpha R})}{nr(n-1+e^{-\alpha R})}\right]. \quad (11)$$

Since each term inside the brackets is negative, Δx_i is always negative. In other words, PLS is a substitute for TS. We derive the change in total savings from

$$\Delta s_i = \frac{1}{\alpha(2+r)}\left[\ln\left(\frac{n-1+e^{-\alpha R}}{n}\right) + \frac{(n-1)(1-e^{-\alpha R})}{n(n-1+e^{-\alpha R})}\right]. \quad (12)$$

We find the following properties for Δs_i when $x_i^o > 0$ and $x_i^e > 0$.

Proposition 3. *Properties of* Δs_i *given interior solutions* ($x_i^o > 0$ *and* $x_i^e > 0$):

1) $\Delta s_i < 0$
2) $\frac{\partial \Delta s_i}{\partial r} > 0$
3) $\frac{\partial \Delta s_i}{\partial R} < 0$
4) $\frac{\partial \Delta s_i}{\partial n} > 0$ *and* $\lim_{n\to\infty} \Delta s_i = 0$

Proof. See Appendix.

For Type-2 TS savers, the reduction in TS deposit outweighs the increase in PLS deposit, and the total savings therefore decreases.

Remark 3 (Type-2 TS Savers). *Assume that all players are identical, i.e.,* $\alpha_i = \alpha$, $\beta_i = \beta$, *and* $\Delta I_i = \Delta I$ *for all i. If* $\beta(1+r) > \frac{ne^{-\alpha(\Delta I - 2y_i^e)}}{n-1+e^{-\alpha R}}$, *all players are Type-2 TS savers. Introducing PLS will decrease both the TS deposit and total savings.*

We summarize the effects of introducing PLS to the status quo (i.e., only a TS account is available) on each type of savings behavior according to Remarks 1 to 3 in Table 1.

Table 1. Effects of Introducing PLS

Type	**Condition**	$\boldsymbol{x_i^o}$	$\boldsymbol{x_i^e}$	$\Delta \boldsymbol{x_i}$	$\Delta \boldsymbol{s_i}$
TS non-savers	$\beta(1+r) \leq e^{-\alpha\Delta I}$	0	0	0	+
Type-1 TS savers	$e^{-\alpha\Delta I} < \beta(1+r) \leq \frac{ne^{-\alpha(\Delta I - 2y_i^e)}}{n-1+e^{-\alpha R}}$	+	0	-	?
Type-2 TS savers	$\beta(1+r) > \frac{ne^{-\alpha(\Delta I - 2y_i^e)}}{n-1+e^{-\alpha R}}$	+	+	-	-

Note: + indicates positive value, - indicates negative value, and ? indicates ambiguous sign.

We illustrate Remarks 1 to 3 in the following example.

Example 5. See Figure 3 for illustration. Assume that the players are identical with $\alpha = 0.5$, $\beta = 0.5$, $I_1 = 1$, and $I_2 = 0$. Equation (9) implies that each player will not save in the status quo (TS non-savers) if $r < 0.213$. If a PLS account is available with $n = 5$ and $R = 1$, then Equation (2) implies that each player will save 0.215 in their PLS account. Equation (7) implies that if $r < 0.633$, each player will not save in her TS account when a PLS account is available. That is, when $0.213 < r < 0.633$, all players are Type-1 TS savers and the effect of introducing PLS is ambiguous—PLS can increase total savings except when r is between 0.606 and 0.633. If $r > 0.633$, each player will save in both TS and PLS accounts (Type-2 TS savers). However, the total savings will be less than x_i^o, which is her TS deposit in the status quo. Notice that the dashed curve representing $x_i^e + y_i^e$ is below the x_i^o curve in Figure 3. Overall, given the set of parameters, PLS increases total savings if and only if r is less than 0.606.

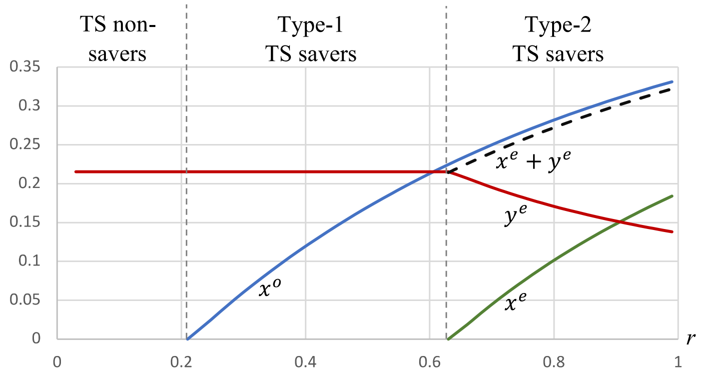


Figure 3. Equilibrium deposits given different TS interest rates (r) in Example 5.

3 Experiment

Based on the theory in the previous section, we conduct within-subject design experiments to test the main theoretical findings.

3.1 Experimental Design

The experiment consists of a series of intertemporal allocations problems. In each scenario, each subject allocates a given endowment of 300 THB into an early payout and a savings amount, which will be paid two weeks and 26 weeks after the experiment, respectively.[17] There are two possible savings options: traditional savings (TS) and prized-linked savings (PLS). All returns from savings will be paid at the same time.

Each subject makes decisions in 34 different scenarios: four TS scenarios where only a TS account is available, six PLS scenarios where only a PLS account is available, and 24 TS+PLS scenarios where both accounts are available. We vary the TS interest rate and the PLS prize across scenarios, as summarized in Table 2.

[17] A daily minimum wage in Thailand was 300 THB (approx. 9 USD) at the time of the experiment.

Table 2. Summary of Scenarios

PLS Prize	TS Interest Rate				
	N/A	**0.25%**	**0.50%**	**0.75%**	**1.00%**
N/A	-	TS 1	TS 2	TS 3	TS 4
1.25	PLS 1	TS+PLS 1	TS+PLS 7	TS+PLS 13	TS+PLS 19
2.5	PLS 2	TS+PLS 2	TS+PLS 8	TS+PLS 14	TS+PLS 20
3.75	PLS 3	TS+PLS 3	TS+PLS 9	TS+PLS 15	TS+PLS 21
5	PLS 4	TS+PLS 4	TS+PLS 10	TS+PLS 16	TS+PLS 22
7.5	PLS 5	TS+PLS 5	TS+PLS 11	TS+PLS 17	TS+PLS 23
10	PLS 6	TS+PLS 6	TS+PLS 12	TS+PLS 18	TS+PLS 24

In the TS scenarios, the interest rates for 26 weeks of savings are 0.25%, 0.50%, 0.75%, and 1.00%, which are comparable to the market interest rates of savings accounts at the time of the experiment. In the PLS scenarios, subjects are randomly assigned into a group of five. In each group, only one subject with a PLS deposit is randomly chosen to win a prize. The prizes in the PLS scenarios are 1.25, 2.5, 3.75, 5, 7.5, and 10 THB. In contrast to the previous studies, our PLS game involves strategic interactions, as the probability of winning is the ratio of the subject's PLS deposit to the total in her group. Since all subjects make simultaneous decisions, the probability of winning is unknown at the time of the decision. If all subjects in a group make no PLS deposits, there will be no winner. In the TS+PLS scenarios, there are 24 scenarios whose parameters are combinations of the four interest rates from the TS scenarios and six PLS prizes from the PLS scenarios.

3.2 Experimental Procedures

We conducted two experimental sessions in November 2017 at the Center for Behavioral and Experimental Economics (CBEE), Chulalongkorn University. We recruited 80 undergraduate students at Chulalongkorn University to participate in one of two sessions, with 40 subjects each. To avoid a possible order effect, we conducted each session with a different order of scenarios. Both sessions were conducted in a computer lab with cubicles using z-Tree (Fischbacher, 2007). We asked each subject to complete a survey at the end of the experiment. The survey included demographic information, opinions and experiences about each savings account, a risk-preference test (similar to Falk et al., 2016), and a Cognitive Reflection test (similar to Frederick, 2005). Descriptive characteristics of the subjects are provided in Table A1 in the appendix. The experimental instructions and the survey are provided in Sections 7.3 and 7.4, respectively.

After the experiment, we randomly selected one scenario to determine each subject's payment. We transferred the early payout and savings amount with returns from savings to each subject's bank account in two weeks and 26 weeks, respectively. We announced the two payment dates at the beginning of the experiment. In addition to these payments, each subject received a 100 THB show-up fee, which was divided equally between the two payments (50 THB each). This is to ensure that, regardless of their allocation decisions, all subjects would receive two payments and could not avoid the additional costs of receiving two separated transfers by allocating an entire endowment in a single payment.

4 Experimental Results

4.1 Total Savings

Table 3. Average Total Savings

PLS Prize	TS Interest Rate				
	N/A	**0.25%**	**0.50%**	**0.75%**	**1.00%**
N/A	-	0.292 (0.353)	0.329 (0.361)	0.406 (0.397)	0.492 (0.427)
1.25	0.299 (0.354)	0.431* (0.421)	0.443* (0.423)	0.461* (0.420)	0.503 (0.415)
2.5	0.309 (0.345)	0.428* (0.408)	0.452* (0.418)	0.454* (0.418)	0.505 (0.419)
3.75	0.349 (0.351)	0.450* (0.419)	0.456* (0.417)	0.473* (0.415)	0.506 (0.417)
5	0.427 (0.367)	0.504* (0.42)	0.491* (0.414)	0.511* (0.418)	0.533 (0.408)
7.5	0.445 (0.367)	0.501* (0.423)	0.527* (0.423)	0.532* (0.427)	0.536 (0.412)
10	0.491 (0.397)	0.570* (0.426)	0.580* (0.426)	0.593* (0.419)	0.617* (0.406)

Notes: 1. Standard deviations are shown in parentheses.
2. The number of samples in each treatment is 80.
3. * indicates that the paired-sample t-test of mean difference between the total savings in TS+PLS scenarios and corresponding TS scenarios (PLS prize is N/A) with the same interest rate rejects the null hypothesis at a 5% significance level.

We commence by analyzing the average savings in each scenario. In all of the following results, we report savings as a proportion of the endowment. Table 3 reports the average total savings in each scenario. Given the same interest rate, the averages of total savings are higher when both TS and PLS accounts are available (TS+PLS scenarios) than when only the TS account is available (TS scenarios).[18] On average, introducing PLS increases the total savings by 12.3 percentage points.[19] These differences in total savings (between 1.1 to 27.8 percentage points) are statistically significant at the 5% level in all scenarios except those with the highest interest rate of 1% and the PLS prize below 10 THB. This is mainly because PLS becomes relatively less attractive when the TS interest rate is high.

To confirm this finding, we estimate a model with the total savings as the dependent variable using OLS. Independent variables include scenario-type dummy variables (TS scenario type is the base category) and subject fixed effects. Model (1) in Table 4 shows that the TS+PLS scenario dummy variable is statistically significant. That is, introducing PLS increases the total savings.

Result 1. *On average, introducing PLS increases the total savings by 12.3 percentage points.*

Table 4. The Treatment Effects on Total Savings

Variable	(1) Total Savings	(2) Total Savings	(3) Change in Total Savings Relative to TS Scenario	(4) Change in Total Savings Relative to PLS Scenario
PLS scenario dummy	0.007	-0.013		
	(0.035)	(0.033)		
TS+PLS scenario dummy	0.123**	0.048		
	(0.028)	(0.031)		
Interest rate		0.159**	-0.180**	0.092*
		(0.030)	(0.048)	(0.038)
PLS prize		0.024**	0.018**	-0.005
		(0.004)	(0.005)	(0.005)
Interest rate × PLS prize		-0.014*	-0.005	-0.005
		(0.004)	(0.005)	(0.005)
R-squared	0.754	0.771	0.650	0.748
Scenarios	All	All	TS+PLS	TS+PLS
Number of observations	2,720	2,720	1,920	1,920

: 1. * and ** indicate significance at the 5% and 1% levels, respectively.
2. Standard errors clustered by subject are shown in parentheses.
3. TS is the base category in columns (1) and (2).
4. All models include constant and subject fixed effects.
5. The change in total savings relative to the TS scenario is equal to the difference between total savings in a TS+PLS scenario and the corresponding TS scenario. The change in total savings relative to the PLS scenario is equal to the difference in total savings in a TS+PLS scenario and the corresponding PLS scenario.

[18] Given the same PLS prize, the averages of the total savings under TS+PLS scenarios are also higher than when only the PLS account is available (PLS scenarios).

[19] This average increase is even bigger than the increase in total savings when the interest rate increases from 0.25% to 0.75% without PLS as an alternative option.

According to Table 3, we observe that the average total savings is increasing in the interest rate and the PLS prize, as expected. Furthermore, we regress the total savings on the scenario dummy variables as well as the interest rate, the PLS prize, and their interaction. As shown in Model (2) in Table 4, the total savings increase with a higher interest rate or a higher PLS prize. Moreover, the negative coefficient of the interaction implies that the effects of the interest rate and the PLS prize are sub-additive.

Importantly, subjects would not make a deposit in any savings account without a return, since the coefficient of the TS+PLS scenario dummy variable becomes statistically insignificant after including the interest rate and the PLS prize. It implies that TS and PLS do not offer any nonfinancial incentives. In other words, subjects increase their total savings when they have an additional savings option because of its monetary benefit, not just because there is one additional option to allocate money.

Next, we analyze how the interest rate and the PLS prize affect the marginal effects of PLS and TS. In Models (3) and (4) in Table 4, we estimate models with the change in total savings relative to the TS scenario and to the PLS scenario, respectively, as the dependent variable. The independent variables are the interest rate, the PLS prize, their interaction terms, and subject fixed effects. We observe a positive relationship between the marginal effect of PLS and the PLS prize as well as between the marginal effect of TS and the interest rate, as expected. In other words, an increase in the total savings is larger when an additional savings option is more attractive.

Since the additional savings option would be less effective if the existing option is relatively attractive, we expect that (1) the marginal effect of PLS is decreasing in the interest rate, and (2) the marginal effect of TS is decreasing in the PLS prize. However, only the former is statistically significant.

Result 2. *Total savings is increasing in both the interest rate and the PLS prize. The marginal effect of PLS is decreasing in the interest rate and increasing in the PLS prize. However, the marginal effect of TS is increasing in the interest rate and independent of the PLS prize.*

4.2 Traditional Savings and Prize-linked Savings

In Table 3, we break down total savings into TS and PLS deposits to see how subjects allocate their endowments in each scenario. In Figure 4, the average TS deposit increases as the interest rate is higher. We also observe a similar relationship between the average PLS deposit and the PLS prize. Furthermore, when available, PLS is apparently a substitute for TS.

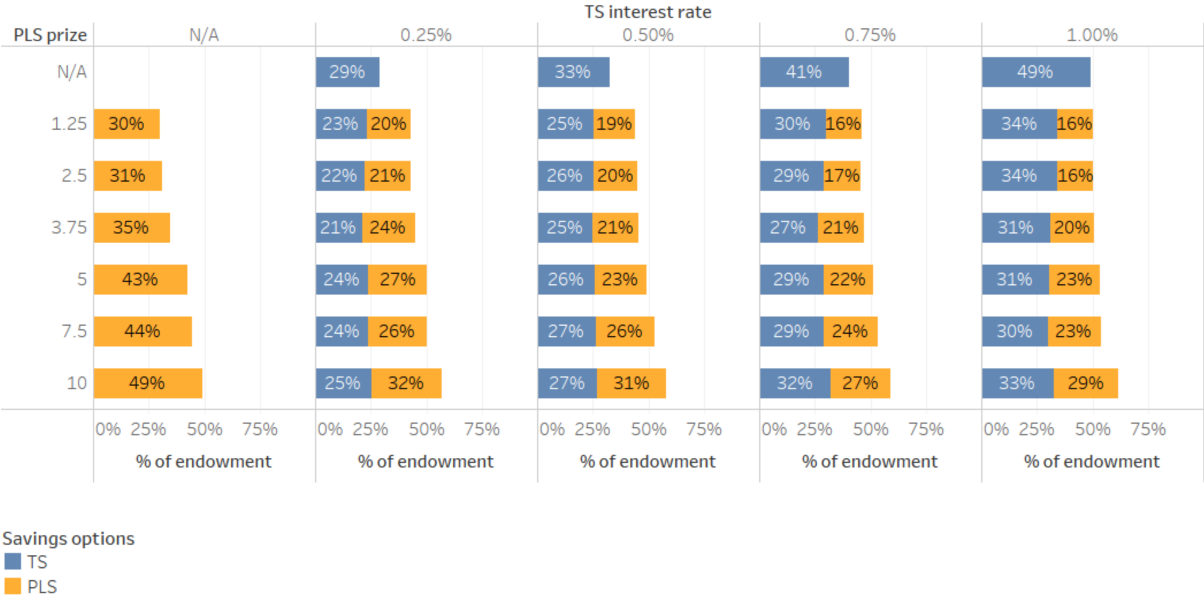


Figure 4. TS and PLS deposits

To gain more insight, we use OLS to estimate Models (5) and (6), in which the dependent variables are the TS deposit and the PLS deposit, respectively, whereas the independent variables are similar to those in Model (2). As shown in Table 5, the PLS prize is statistically significant and positive in Model (5). This implies that an introduction of PLS leads to an increase in the TS deposit when the interest rate is lower than 0.74%.[20] This result contradicts the theory that posits a TS and PLS substitution. In contrast, Model (6) shows that the interest rate does not statistically affect the PLS deposit.

Result 3. *When the interest rate is low, a higher PLS prize induces the TS deposit, suggesting complementarity between the two types of savings. However, the interest rate has no significant effect on PLS deposits.*

[20] According to Model (5), the marginal effect of the PLS prize on TS deposits is $0.010 - 0.013 \cdot$ interest rate. So, the marginal effect is positive when the interest rate is smaller than 0.74%.

Table 5. The Treatment Effects on TS and PLS Rates

Variable	(5) TS	(6) PLS
TS+PLS scenario set	-0.111**	-0.128**
	(0.032)	(0.038)
Interest rate	0.199**	-0.017
	(0.340)	(0.051)
PLS prize	0.010*	0.019**
	(0.004)	(0.004)
Interest rate × PLS prize	-0.013**	-0.007
	(0.005)	(0.005)
R-squared	0.733	0.570
Scenarios	TS and TS+PLS	PLS and TS+PLS
Number of observations	2,240	2,400

Notes: 1. * and ** indicate significance at the 5% and 1% levels, respectively.
2. Standard errors clustered by subject are shown in parentheses.
3. TS and PLS scenarios are the baseline in Models (5) and (6), respectively.
4. Both models include constant and subject fixed effects.

4.3 Effect of PLS on Individuals

In the experiment, we observe many extreme decisions, non-saver and all-saver, which involve allocating all of the endowment either to the early or the delayed payments. The numbers of these extreme decisions change after introducing TS or PLS as an additional savings option, yet to a different extent. As shown in Table 6, 39% of decisions are non-saver when only the TS account is available. When both TS and PLS accounts are available, this percentage significantly falls to 23% ($t = 6.18, p < 0.01$). On the other hand, introducing TS does not significantly change the proportion of non-saver decisions (24% in PLS scenarios versus 23% in TS+PLS scenarios, $t = 0.46, p = 0.65$). In addition, the percentage of all-saver decisions is significantly higher in TS+PLS scenarios when compared to TS and PLS scenarios ($t = 5.71, p < 0.01$ and $t = 5.22, p < 0.01$, respectively).

Table 6. Proportions of Non-Saver and All-Saver Decisions

	TS		PLS		TS+PLS	
	Obs.	%	Obs.	%	Obs.	%
Non-saver	125	39.1*	115	24.0	441	23.0
(0% savings)		(0.489)		(0.427)		(0.421)
All-saver	51	15.9*	82	17.1*	576	30.0
(100% savings)		(0.337)		(0.377)		(0.458)
Total obs.	320		480		1,920	

Notes: 1. Standard deviations are shown in parentheses.
2. * indicates that the t-test rejects the null hypothesis that the percent of decisions in the TS or PLS scenario set is different from the TS+PLS scenario set at a 5% significance level.

In the experiment, we observe many extreme decisions, non-saver and all-saver, which involve allocating all of the endowment either to the early or the delayed payments. The numbers of these extreme decisions change after introducing TS or PLS as an additional savings option, yet to a different extent. As shown in Table 6, 39% of decisions are non-saver when only the TS account is available. When both TS and PLS accounts are available, this percentage significantly falls to 23% ($t = 6.18, p < 0.01$). On the other hand, introducing TS does not significantly change the proportion of non-saver decisions (24% in PLS scenarios versus 23% in TS+PLS scenarios, $t = 0.46, p = 0.65$). In addition, the percentage of all-saver decisions is significantly higher in TS+PLS scenarios when compared to TS and PLS scenarios ($t = 5.71, p < 0.01$ and $t = 5.22, p < 0.01$, respectively).

Result 4. *Introducing a TS or PLS account significantly increases the percentage of all-saver decisions by 12.9 and 14.1 percentage points, respectively. However, only the PLS account significantly reduces the percentage of non-saver decisions (by 16.1 percentage points).*

Table 7. Effects of PLS on Savings Decisions in TS+PLS Scenarios

TS Deposit	PLS Deposit	Change in Total Savings			Total	
		Increase	**Unchanged**	**Decrease**	**Obs.**	**%**
TS non-saver						
0	0	N/A	408	N/A	408	54.4
0	+	212	N/A	N/A	212	28.3
+	0	84	N/A	N/A	84	11.2
+	+	46	N/A	N/A	46	6.1
Total		342	441	N/A	750	100.0
TS saver						
0	0	N/A	N/A	33	33	2.8
0	+	21	45	50	116	9.9
+	0	27	144	21	192	16.4
+	+	484	193	152	829	70.9
Total		532	382	223	1,170	100.0
Total TS decisions		874	823	223	1,920	

In Table 7, we separate the decisions in TS scenarios into two groups, TS non-saver (no TS deposit) and TS saver (positive TS deposit), and report the directions of changes in total savings after introducing PLS for each group. For TS non-savers, there are 212 decisions (28.3%), with only a PLS deposit when PLS is available, consistent with our theoretical prediction in Remark 1. However, after introducing PLS, the majority of TS non-saver decisions (408, or 54.4%) still involve no deposits in either account. This result contradicts Remark 1 but could be explained by heterogeneous preferences. As illustrated in Example 4, more risk-averse subjects may avoid PLS in equilibrium, given beliefs that others are less risk-averse. There are 84 TS non-saver decisions (11.2%) that save exclusively in TS when PLS becomes available; they clearly violate the independence of irrelevant alternatives (IIA) condition. Lastly, there are 46 decisions (6.1%) that involve savings in both TS and PLS accounts. These could be a result of a complementarity between the two accounts or a violation of the IIA condition.

Among TS saver decisions, we find a small number of decisions (33, or 2.8%) with no savings when PLS is available. These decisions also violate the IIA condition. According to Remarks 2 and 3, there are two groups of TS saver decisions: Type-1 and Type-2. Type-1 TS savers will save only in PLS, while Type-2 TS savers will save in both accounts. We find 116 Type-1 TS saver decisions (9.9%) and 829 Type-2 TS saver decisions (70.9%). However, the majority of Type-2 TS saver decisions (484 out of 829) display an increase in total savings, contradicting the theoretical prediction of lower total savings in Remark 3. Only 152 decisions are consistent with this prediction. There are 192 decisions (16.4%) with only the TS deposit even when PLS is available. Heterogeneous preference can also explain these decisions (see Example 4).[21]

Result 5. *Contradicting Remark 3, more than half of all decisions with both TS and PLS deposits increase the total savings when PLS is available.*

4.4 Average Deposits by Subject's Preference on Savings Account

We investigate each subject's self-reported preference on TS and PLS.[22] In the exit survey, subjects state whether they prefer TS or PLS or feel indifferent. Table 8 reports the average deposit for each response. Subjects who prefer PLS have the lowest savings in all scenarios. Even when PLS is the only option, the average deposit is more than 10 percentage points lower than in the other two groups. In each group, the average deposit is higher in TS+PLS scenarios than that in TS scenarios.

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[21] We also separately estimate the treatment effect of introducing PLS to TS non-savers and TS savers. Introducing PLS has a larger impact on TS non-savers, as the increases in total savings for TS non-savers and TS savers are 19.9 and 7.4 percentage points, respectively (see Table A3 in the appendix).

[22] We also investigate the role of observable characteristics in explaining the change in total savings relative to TS or PLS scenarios; the most observable characteristics could not explain the results. We report our findings in Table A2 in the appendix.

Table 8. Average Deposits by Subject's Preference on Savings Account

Preference	TS scenarios	PLS scenarios	TS+PLS scenarios		
			TS	PLS	Total
Prefer TS	0.507	0.418	0.402	0.204	0.607
	(0.382)	(0.339)	(0.386)	(0.259)	(0.391)
Prefer PLS	0.239	0.291	0.131	0.243	0.374
	(0.351)	(0.346)	(0.204)	(0.316)	(0.401)
Indifferent	0.339	0.459	0.243	0.244	0.487
	(0.391)	(0.425)	(0.322)	(0.317)	(0.440)
All subjects	0.380	0.387	0.276	0.226	0.503
	(0.392)	(0.369)	(0.342)	(0.293)	(0.419)

Notes: 1. Standard errors are shown in parentheses.
2. 35 subjects prefer TS, 26 subjects prefer PLS, and 19 subjects are indifferent.

When both savings options are available, the average total savings by subjects who prefer TS is the highest, at 61%, while those who prefer PLS save only 37%. We observe a large difference in the average proportion of PLS deposit to total savings across the three groups—two-thirds for those who prefer PLS, one-half for those who are indifferent, and only one-third for those who prefer TS.

Result 6. *PLS can effectively increase the total savings for all groups, but the average proportion of PLS deposit to total savings is highest among subjects who prefer PLS.*

5 Conclusion

We develop a game-theoretic model of portfolio allocation decisions, where traditional savings (TS) and prize-linked savings (PLS) are available, and experimentally examine the effectiveness of PLS in the laboratory. Our experimental results show that introducing PLS increases average total savings by 12 percentage points. This positive effect is particularly strong among subjects with low TS deposits. Introducing PLS reduces the proportion of non-savers from 39% to 23% and induces total savings among the non-savers by 20 percentage points.

Whether these results have external validity remains an open question. Our findings are based on situations with up to two savings options, whereas, in the real world, the line between savings and investment is blurry and individuals may consider several alternatives in their portfolio allocation problem. A future study may try to address this issue and test the robustness of our results.

While our experimental findings are consistent with previous studies, our game-theoretic approach enables us to make further implications about PLS organized on a smaller scale, where strategic interaction among savers matters. For example, policy makers can design a local PLS program that fits the target population and campaign budget. How to design and implement PLS products in various settings is an intriguing area for future research.

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7 Appendix

7.1 Proofs

Proof of Proposition 1:

We find that (i) $s_i \in [0,1)$, (ii) $s_i \to 1$ as $\psi \to 0$, (iii) $s_i \to 0$ as $\psi \to \kappa_i$, and (iv) $\frac{ds_i}{d\psi} = \frac{-\alpha_i r e^{-\alpha_i R}}{(1-e^{-\alpha_i R})(1-\alpha_i r\psi)^2} < 0$ for all $\psi \in (0, \kappa_i)$. It follows that (i) $\sum_{i=1}^{n} s_i \in [0, n)$, (ii) $\sum_{i=1}^{n} s_i \to n$ as $\psi \to 0$, (iii) $\sum_{i=1}^{n} s_i \to 0$ as $\psi \to \hat{\kappa}$ where $\hat{\kappa} := \max \kappa_i$, and (iv) $\sum_{i=1}^{n} s_i$ is strictly decreasing for all $\psi \in (0, \hat{\kappa})$. Thus, there exists a unique value of $\psi \in (0, \hat{\kappa})$ such that $\sum_{i=1}^{n} s_i$, i.e., the sum of probabilities of winning, is equal to 1. ■

Proof of Proposition 2:

1) Given $y_i^e = \frac{(n-1)(1-e^{-\alpha R})}{\alpha n r(n-1+e^{-\alpha R})}$, we find $\frac{\partial y_i^e}{\partial \alpha} = \frac{(n-1)}{nr}\left[\frac{n(\alpha R e^{-\alpha R}-1+e^{-\alpha R})+(1-e^{-\alpha R})^2}{\alpha^2(n-1+e^{-\alpha R})^2}\right]$. Let $n = 2$. The numerator inside the brackets can be written as $2(\alpha R e^{-\alpha R} - 1 + e^{-\alpha R}) + (1 - e^{-\alpha R})^2 = 2\alpha R e^{-\alpha R} - 1 + e^{-2\alpha R}$. We define $F(a) = e^{a} - a e^{a/2}$. We know that $F(0) = 1$, $F'(0) = 0$, and $F'(a) > 0$ for all $a \neq 0$. Thus $F(a) < 1$ for all $a < 0$. It follows that $F(-2\alpha R) = e^{-2\alpha R} + 2\alpha R e^{-\alpha R} < 1$, which is equivalent to $2\alpha R e^{-\alpha R} - 1 + e^{-2\alpha R} < 0$. Thus $\frac{\partial y_i^e}{\partial \alpha} < 0$ for $n = 2$. Since $2(\alpha R e^{-\alpha R} - 1 + e^{-\alpha R}) + (1 - e^{-\alpha R})^2 < 0$, it is necessary that $\alpha R e^{-\alpha R} - 1 + e^{-\alpha R} < 0$. It follows that $n(\alpha R e^{-\alpha R} - 1 + e^{-\alpha R}) + (1 - e^{-\alpha R})^2 < 0$ for all $n > 2$. Therefore, $\frac{\partial y_i^e}{\partial \alpha} < 0$ for any $n \geq 2$.

2) $\frac{\partial y_i^e}{\partial r} = -\frac{(n-1)(1-e^{-\alpha R})}{\alpha n r^2(n-1+e^{-\alpha R})} < 0$.

3) $\frac{\partial y_i^e}{\partial R} = \frac{(n-1)e^{-\alpha R}}{r(n-1+e^{-\alpha R})^2} > 0$.

4) $\frac{\partial y_i^e}{\partial n} = -\frac{(1-e^{-\alpha R})}{\alpha r}\left[\frac{(1-e^{-\alpha R})+n(n-2)}{n^2(n-1+e^{-\alpha R})^2}\right] < 0$. Since Eq (3) implies $y_i^e = \frac{(1-e^{-\alpha R})}{\alpha r[e^{-\alpha R}n/(n-1)+n]}$, it follows immediately that $\lim_{n\to\infty} y_i^e = 0$. ■

Proof of Proposition 3:

1) For $n = 2$, we have $\Delta s = \frac{1}{\alpha(2+r)}\left[\ln\left(\frac{1+e^{-\alpha R}}{2}\right) + \frac{(1+e^{-\alpha R})}{2(1+e^{-\alpha R})}\right]$. We define $G(a) = 2\ln a + \frac{1}{a}$. We know that $G(a) < 1$ for all $a \in (\frac{1}{2}, 1)$. Let $a = \frac{1+e^{-\alpha R}}{2}$. Then, $2\ln\left(\frac{1+e^{-\alpha R}}{2}\right) + \frac{2}{1+e^{-\alpha R}} < 1$ which is equivalent to $\ln\left(\frac{1+e^{-\alpha R}}{2}\right) + \frac{(1-e^{-\alpha R})}{2(1+e^{-\alpha R})} < 0$. Thus,

$\Delta s < 0$ given $n = 2$. In part 4 below, we prove that $\frac{\partial \Delta s}{\partial n} > 0$, and $\lim_{n\to\infty} \Delta s = 0$. Therefore, $\Delta s < 0$ given any $n \geq 2$.

2) Given Eq (5), we find that $\frac{\partial \Delta s}{\partial r} = -\frac{1}{\alpha(2+r)^2}\left[\ln\left(\frac{n-1+e^{-\alpha R}}{n}\right) + \frac{(n-1)(1-e^{-\alpha R})}{n(n-1+e^{-\alpha R})}\right] < 0$.

3) Given Eq (5), we find that $\frac{\partial \Delta s}{\partial R} = -\frac{e^{-2\alpha R}}{(2+r)(n-1+e^{-\alpha R})^2} < 0$.

4) Given Eq (5), we find that $\frac{\partial \Delta s}{\partial n} = \frac{(1-e^{-\alpha R})[n-1+(n+1)e^{-\alpha R}]}{\alpha(2+r)n^2(n-1+e^{-\alpha R})^2} > 0$. Given Δ in Eq (5), we find that each of the two terms in the brackets converges to 0 as n goes to ∞. It follows immediately that $\lim_{n\to\infty} \Delta s = 0$. ■

7.2 Additional Tables

Table A1. Descriptive Characteristics of Subjects

Characteristics	Percentage of Subjects	Characteristics	Percentage of subjects
Gender		Savings rate	
Male	0.34	< 10%	0.20
Female	0.66	10–20%	0.31
Age		20–30%	0.26
18–19	0.14	30–40%	0.16
20–21	0.61	> 40%	0.06
22–24	0.25	Lottery	
Department		Never	0.64
Economics	0.21	Once in a while	0.30
Science	0.19	Few times a year	0.05
Business	0.15	Every month	0.00
Pharmacy	0.14	Every week	0.01
Engineering	0.06	Gamble	
Arts	0.06	Never	0.78
Psychology	0.06	Once in a while	0.16
Other	0.13	Few times a year	0.06
Year		Every month	0.00
First	0.13	Every week	0.00
Second	0.20	Has bought PLS	
Third	0.33	Yes	0.28
Fourth or above	0.24	No	0.72
Cognitive score			
0	0.15		
1	0.14		
2	0.30		
3	0.41		

Notes: 1. The number of observations is 80.
2. Other departments include Architecture, Communications, Education, Political Science, Sport Science, Veterinary Science, and Health Science.

Table A2. Effects of Observable Characteristics on Incremental Savings from PLS

Variable	(1) OLS	(2) OLS	(3) Random Intercept
Interest rate	-0.180**	-0.180**	-0.180**
	(0.047)	(0.047)	(0.047)
PLS prize	0.018**	0.018**	0.018**
	(0.006)	(0.005)	(0.005)
Interest rate × PLS prize	-0.005	-0.005	-0.005
	(0.008)	(0.005)	(0.005)
Female	0.043**	0.043	0.043
	(0.015)	(0.053)	(0.052)
Savings rate	0.170**	0.170	0.170
	(0.059)	(0.216)	(0.216)
Lottery	0.017	0.017	0.017
	(0.010)	(0.027)	(0.027)
Cognitive score	0.048**	0.048*	0.048*
	(0.007)	(0.022)	(0.022)
Switching point (risk)	7.967**	7.967	7.967
	(1.071)	(4.268)	(4.257)
Switching point (time)	-2.518**	-2.518	-2.518
	(0.756)	(2.354)	(2.348)
Variance of constant	-	-	0.051**
			(0.015)
R-squared	0.118	0.118	-
Log pseudolikelihood	-	-	330.134
Number of observations	1,920	1,920	1,920

Notes: 1. The dependent variable in all models is the difference in total savings between the TS and TS+PLS scenarios.
2. * and ** indicate significance at the 5% and 1% levels, respectively.
3. Standard errors clustered by subject are shown in parentheses.
4. PLS and TS are the baseline treatments in Models (5) and (6), respectively.
5. All models include constant and subject fixed effects.

Table A3. The Treatment Effects on Total Savings

Variable	(1) TS Non-Savers	(2) TS Savers
TS+PLS scenario dummy	0.199*	0.074*
	(0.053)	(0.025)
R-squared	0.660	0.794
Scenarios	TS, TS+PLS	TS, TS+PLS
Number of observations	875	1,365

Notes: 1. The dependent variable in both models is the total savings.
2. * and ** indicate significance at the 5% and 1% levels, respectively.
3. The TS scenario is the baseline in both models.
3. Standard errors clustered by subject are shown in parentheses.
4. All models include constant and subject fixed effects.

7.3 Instructions

Welcome to experiments on decision making. Your compensation for participating in this experiment will be transferred to your SCB bank account as indicated on the compensation form. The compensation consists of the following:

1) The fee for participation and survey response is 100 THB. You will receive half of the fee (50 THB) two weeks from today and the remaining half (50 THB) 26 weeks from today.

2) Payment from your decision is at least 300 THB. You will receive the payment in two weeks or 26 weeks from today, or some payment in two weeks and the remaining in 26 weeks. The payment amounts in two and 26 weeks depend on your decisions in the experiment.

Today is ______________________.

First payment (2 weeks from today) is on ______________________.

Second payment (26 weeks from today) is on ______________________.

This experiment consists of two parts. The first part is the experiment on decision making, which will take less than one hour. The second part is the questionnaire, which will take less than 30 minutes.

Part 1 Experiment on Decision Making

The first part consists of three sets of scenarios in which you need to make a decision. There are 34 scenarios for you to make a decision. One of the 34 scenarios will be randomly selected to calculate your payment at the end of the session. Since you do not know which scenario will be selected, you should make a careful decision in every scenario. The details of scenario sets A to C are as follows.

Scenario Set A

Scenario Set A consists of five scenarios: A1 to A4. In each scenario, you need to divide 300 THB into two piles: early payment and Savings AA deposit. You will receive the early payment in two weeks from today and your Savings AA deposit plus an interest in 26 weeks from today. The interest rates are different across scenarios. For example, in Scenario A1 where the interest rate is 0.25%, you choose to receive early payment of 200 THB and deposit 100 THB in savings AA. You will receive a total compensation of 50 + 200 = 250 THB in the first payment and 50 + 100 + 0.25 = 150.25 in the second payment (0.25% interest of 100 THB deposit is equal to 0.25 THB).

On your computer screen, you need to choose the deposit amount in Savings AA. The early payment will be automatically calculated as 300 less the deposit amount. The screen will show the first and second payments. Please make your desired decisions in scenarios A1 to A4.

Scenario Set B

Scenario Set B consists of 6 scenarios: B1 to B6. In each scenario, you need to divide 300 THB into two piles: early payment and Savings BB deposit. You will receive the early payment in two weeks from today and your Savings B deposits plus returns in 26 weeks from today. Savings BB yields no interest but you will have a chance to win a prize. The chance to win a prize depends on your and other participants' deposits. Specifically, you will be randomly group with other four participants. One of the five participants in each group will win a prize. The chance of winning a prize is your share of the total Savings BB deposit in your group. You will not know whom you are grouped with.

Here is an example. If you deposit 100 THB in Savings BB and the total Savings BB deposit in your group is 400 THB, you will have 100 ÷ 400 = 25% chance to win a prize. If instead the total Savings BB deposit in your group is 1,000 THB, you will have 100 ÷ 1,000 = 10% chance to win a prize. You will have more chance to win a prize if you deposit more in Savings BB. Prizes of Savings BB are different across scenarios. Please make your desired decisions in scenarios B1 to B6.

Scenario Set C

Scenario Set C consists of 24 scenarios: C1 to C24. In each scenario, you need to divide 300 THB into three piles: early payment, Savings AA deposit and Savings BB deposit. You will receive the early payment in two weeks from today and your Savings AA and Savings BB deposits plus their returns in 26 weeks from today. Savings AA payouts an interest as in Scenario Set A. Savings BB yields no interest but you will have a chance to win a prize as in Scenario Set B. You will have more chance to win a prize if you deposit more in Savings BB. The interest rates for Savings AA and prizes for Savings BB are different across the scenarios. Please make your desired decisions in scenarios C1 to C24.

7.4 Survey Questions

1) Age
2) Gender
3) Department
4) Year
5) In Scenario Set A, how do you decide how to divide the money between the first payment in two weeks and the second payments in 26 weeks?
6) What is your opinion on the savings that yields no interest but provides a chance to win a prize?
7) Between Savings AA, which pays a fixed interest rate, and Savings BB, which yields no interest but provides a chance to win a prize, which one do you prefer? (a) Savings AA, (b) Savings BB, or (c) Indifferent.
8) What percent of your monthly allowance or salary do you save?
9) What is the total amount you saved last month?
10) Have you bought into the Government Savings Bank's or Bank for Agriculture and Agricultural Cooperatives' lotteries?
11) Do you currently have Government Savings Bank's or Bank for Agriculture and Agricultural Cooperatives' lottery tickets that have not been matured?
12) How frequent do you buy lottery tickets? (a) every round, (b) every month, (c) a few times a year, (d) once in a while, or (e) never.
13) How frequently do you gamble, such as soccer betting, card gambling, and casino gambling? (a) every week, (b) every month, (c) a few times a year, (d) once in a while, or (e) never.

<u>Cognitive Reflection Test</u>

1) A bat and a ball cost $1.10 in total. The bat costs $1.00 more than the ball. How much does the ball cost?
2) If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?
3) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?