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A natural MSSM from a novel SO(10), Yukawa unification, light sparticles, and SUSY implications at LHC

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Abstract: The $\mathsf{SO}(10)$ model with a heavy Higgs spectrum consisting of $\mathsf{560}+\overline{\mathsf{560}}$ and a light Higgs spectrum consisting of $2\times\mathsf{10}+\mathsf{320}$ plet representations of $\mathsf{SO}(10)$ is unique among $\mathsf{SO}(10)$ models. It has the remarkable property that VEVs of $\mathsf{560}$ and $\overline{\mathsf{560}}$ can simultaneously reduce the rank of the gauge group and further reduce the remaining symmetry down to the Standard Model gauge group. Additionally, on mixing with the light fields all the Higgs fields become heavy except for one pair of light Higgs doublets just as in MSSM. This model has not been fully explored thus far because of the technical difficulty of computing the couplings of the heavy and the light Higgs sectors, specifically the interaction $(\mathsf{560}\times\mathsf{560})\cdot\mathsf{320}$ involving the coupling of tensor-spinors with a third rank mixed tensor $\mathsf{320}$. An explicit analysis of such couplings is given in this paper. Spontaneous symmetry breaking of the $\mathsf{SO}(10)$ symmetry is carried out by reducing the gauge group to $\mathsf{SU}(3)_c\times\mathsf{SU}(2)_L\times\mathsf{U}(1)_Y$ with just one pair of light Higgs. Thus a natural deduction of MSSM arises from the $\mathsf{SO}(10)$ model with no fine tuning needed. Further, it is shown that the light Higgs doublet of the model is a linear combination of the Higgs doublet fields of the $2\times\mathsf{10}$ and the $\mathsf{320}$ Higgs fields. It is shown that in this class of $\mathsf{SO}(10)$ models $b-t-\tau$ unification can be achieved with $\tan\beta$ as low as 5–10. An analysis of the sparticle spectrum within $\tilde{g}$SUGRA renormalization group evolution is given which leads to a bi-modal sparticle spectrum consisting of a compressed low mass spectrum for sleptons and weakinos and a high mass spectrum of gluino, squarks, and heavy Higgs. While the LSP is the light neutralino, the NLSP is found to be the light stau lying close to the LSP, while the remaining leptons, and the weakinos are also in close proximity to the LSP with masses in the few hundred GeV range. The cross section for slepton production and weakino production are estimated and appear promising for SUSY at the LHC. However, a more dedicated analysis is needed to predict the size of the supersymmetric signatures at the LHC.

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JHEP07(2025)222

Contents

1 Introduction 2

2 The model 5

3 One step breaking of $\mathsf{SO}(10)$ to $\mathsf{SU}(3)_\mathsf{C} \times \mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y}$ and natural doublet-triplet splitting 7

4 The Higgs doublet mass matrix 8

5 Numerical analysis of breaking of $\mathsf{SO}(10)$ to $\mathsf{SU}(3)_\mathsf{C} \times \mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y}$ and generation of light Higgs doublets 10

6 Yukawa unification, light sparticles, and SUSY implications at LHC 11

7 Conclusion 16

A $\mathsf{560} + \overline{\mathsf{560}}$ tensor-spinor in its $\mathsf{SU}(5)$ oscillator modes 22
- A.1 The reducible 720 dimensional tensor-spinor multiplet 22
- A.2 Normalization of kinetic energies of irreducible $\mathsf{SU}(5)$ multiplets arising in the decomposition of $\mathsf{560} + \overline{\mathsf{560}}$ multiplets 26

B Couplings of the $\mathsf{SU}(5)$ components 1, 24, 75 that enter in spontaneous breaking of $\mathsf{SO}(10)$ GUT symmetry 27

C Extraction and normalization of $\mathsf{SU}(2)_\mathsf{L}$ doublet fields in the model 29
- C.1 $\mathsf{5} + \overline{\mathsf{5}}$ and $\mathsf{45} + \overline{\mathsf{45}}$ of $\mathsf{SU}(5)$ 29
- C.2 $\mathsf{70} + \overline{\mathsf{70}}$ of $\mathsf{SU}(5)$ 30

D The 320 dimensional tensors of $\mathsf{SO}(10)$ 31
- D.1 Construction of $\Lambda^{(320_a)}_{[\mu\nu]\lambda}$ tensor 31
- D.2 Construction of $\Lambda^{(320_s)}_{(\mu\nu)\lambda}$ tensor 32

E Possible trilinear couplings of the 320-plet with the 560-plet and their generational symmetries 33
- E.1 Couplings that involve three gammas 33
- E.2 Couplings that involve a single gamma 34

F Construction of normalized 70-, 45- and 5-plets with diagonal kinetic energy terms contained in $\Lambda^{320_s}_{(\mu\nu)\lambda}$ of $\mathsf{SO}(10)$ 35
- F.1 STEP 1: identifying three 70's, two 45's and eight 5's in $\Lambda^{320_s}_{(\mu\nu)\lambda}$ 35
- F.2 STEP 2: construction of the kinetic energy of $\Lambda^{320_s}_{(\mu\nu)\lambda}$ multiplet 37
- F.3 General case of normalization of the kinetic energy after diagonalization 37

F.4 STEP 3: diagonalization and normalization of kinetic energy of 70, 45 and 5 multiplets 38

G Details of doublet couplings **42**
G.1 Computation of $560 \cdot 560 \cdot 320_s$ and $\overline{560} \cdot \overline{560} \cdot 320_s$ interactions 42
G.2 Computation of $560 \cdot 560 \cdot 10_1$, $\overline{560} \cdot \overline{560} \cdot (10_1 + 10_2)$ and $560 \cdot \overline{560}$ interactions 46

1 Introduction

Among grand unified theories $SO(10)$ unification is the most appealing since aside from unifying the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ it also accommodates a full generation of quarks and leptons in one 16-plet spinor representation of $SO(10)$ [1, 2] (for a review of grand unified theories see, e.g. [3]). There is a further feature of $SO(10)$ unification which sets it apart from the minimal unifying gauge group $SU(5)$. Thus in $SU(5)$, just one 24-plet of Higgs breaks the GUT symmetry down to the Standard Model gauge group. For $SO(10)$ there exist a variety of possibilities with both small as well large representations (see, e.g., [4–11] and for applications see, e.g., [12–18]). Additionally unlike $SU(5)$ we need to reduce the rank of the gauge group. To check which Higgs representation can produce rank reduction we look at the $SU(5) \times U(1)$ quantum numbers of some possible candidate tensor representations, such as 45, 54, 210. Their $SU(5) \times U(1)$ decomposition are: $45 = 1(0) + 10(4) + \overline{10}(-4) + 24(0)$, $54 = 15(4) + \overline{15}(-4) + 24(0)$ and $210 = 1(0) + 5(-8) + \overline{5}(8) + 10(4) + \overline{10}(-4) + 24(0) + 40(-4) + \overline{40}(4) + 75(0)$. The $SU(5)$ representations whose VEVs preserve the Standard Model gauge group are $1(0)$, $24(0)$ and $75(0)$ but all of these have a zero $U(1)$ quantum number and thus cannot reduce the rank. A possible Higgs representation which can reduce the rank is the spinor representation 16 which under $SU(5) \times U(1)$ has the decomposition $16 = 1(-5) + \overline{5}(-3) + 10(-1)$ where the singlet has a non-vanishing $U(1)$ quantum number. Thus in this case the minimal Higgs representation which can break $SO(10)$ down to Standard Model gauge group is $16 + \overline{16} + 45$. Another possibility for reducing the rank is to use 126-plet of Higgs which under $SU(5) \times U(1)$ has the decomposition $126 = 1(-10) + \overline{5}(-2) + 10(-6) + \overline{15}(6) + 45(2) + \overline{50}(-2)$. Here the singlet field has a non-vanishing $U(1)$ quantum number and can reduce the rank. Thus in this case using $45 + 126 + \overline{126}$ will break $SO(10)$ to the Standard Model gauge group. However, we further need to break the symmetry down to $SU(3)_C \times U(1)_{em}$ which requires a 10-plet of Higgs which under $SU(5) \times U(1)$ has the decomposition $10 = 5(2) + \overline{5}(-2)$ and has Higgs doublets. The discussion above illustrates that the breaking of $SO(10)$ to $SU(3)_C \times U(1)_{em}$ requires three sets of Higgs representations, i.e., one to reduce the rank, the second to break the rest of the symmetry down to the standard model gauge group and the third to break it further to the residual symmetry $SU(3)_C \times U(1)_{em}$. A while back another proposal was made where the combination presentations $144 + \overline{144}$ was used to break the $SO(10)$ GUT symmetry [19, 20]. Under $SU(5) \times U(1)$ decomposition [21] $144 = \overline{5}(3) + 5(7) + 10(-1) + 15(-1) + 24(-5) + 40(-1) + \overline{45}(3)$. Here since $24(-5)$ has a non-vanishing $U(1)$ quantum number, a VEV of $24(-5)$ will reduce the rank and then break the rest of the symmetry of $SO(10)$ down to the Standard Model gauge group just in one step.

JHEP07(2025)222

The $\mathsf{144}+\overline{\mathsf{144}}$ model discussed above, however, suffers from the problem typical of most grand unified models. Thus, as well known, typically in grand unified models the Higgs doublets would turn out to be the same size in mass as the Higgs triplets which are required to be heavy to suppress rapid proton decay (see, e.g., [3]). There are a variety of proposals on how to produce a light Higgs doublet needed for electroweak symmetry breaking. One such proposal is to use fine tuning to force one Higgs doublet pair to be light. However, such a procedure is viewed unnatural since the fine tuning involved is extreme. There are various other possibilities where a light pair of doublets could arise from a symmetry principle, or a group theoretic constraint or a more fundamental aspect of the theory itself. Such models may be viewed as natural. Models of this type exist in strings, see, e.g., [22–25]. One of the early ways to get light Higgs doublets while keeping the Higgs color triplets heavy is to arrange the vacuum expectation value (VEV) so that the spontaneous breaking of SO(10) occurs only along the $B-L$ preserving direction (see, e.g., [26–28]). Thus, if the SO(10) symmetry breaking occurs along the $B-L$ preserving direction, then the Higgs color triplets will get masses and the Higgs doublets will remain light.

There is yet another mechanism for naturally generating a doublet-triplet splitting. It is the well known missing partner mechanism which does not work for SU(5) but is feasible in SO(10) model building. This mechanism involves two Higgs sectors: one heavy and the other light. Here the choice of heavy vs light Higgs representations is such that there is an exact match between the number of heavy Higgs color triplets/anti-triplets and light Higgs color triplets/anti-triplets but there is a mismatch in the number of heavy Higgs doublet pairs n_h and the number of light Higgs doublet pairs n_l such that $n_l = n_h + 1$. In this case in the presence of mixing of light and heavy Higgs fields, all Higgs triplets/anti-triplets will become heavy while only $2n_h$ number of Higgs doublet pairs will become heavy leaving one Higgs doublet pair light. A key element of this construction is the requirement that there exist a heavy Higgs representation where the number of Higgs doublet pairs is one less than the number of Higgs triplet-anti-triplet pairs. An example of such representations in SU(5) is the combination $\mathsf{50}+\overline{\mathsf{50}}$-plets of heavy Higgs which has one pair of heavy Higgs triplets/anti-triplets and there are no Higgs doublets. In this case if one chooses the heavy sector to be $\mathsf{50}+\overline{\mathsf{50}}+\mathsf{75}$ and the light sector to be $\mathsf{5}+\overline{\mathsf{5}}$, the 75-plet induces a mixing between the heavy $\mathsf{50}+\overline{\mathsf{50}}$ Higgs and $\mathsf{5}+\overline{\mathsf{5}}$ light Higgs, generating heavy masses for all the Higgs triplets/anti-triplets and leaving one Higgs doublet pair light [29, 30]. An example of such a model in SO(10) was discussed in [10] where the heavy sector consists of $\mathsf{126}+\overline{\mathsf{126}}+\mathsf{210}$ and the light sector consists of $2\times\mathsf{10}+\mathsf{120}$, and the $\mathsf{126}+\overline{\mathsf{126}}$ contain $\mathsf{50}+\overline{\mathsf{50}}$ providing the key element for the missing partner mechanism to work in SO(10). A more exhaustive analysis of the missing partner mechanism in SO(10) was given in [31] where several other models producing a light Higgs doublet pair were discussed.

Of specific interest for the work in this paper is the model discussed in [31] consisting of

$$\text{Heavy }\{\mathsf{560}+\overline{\mathsf{560}}\}+\text{Light }\{2\times\mathsf{10}+\mathsf{320}\} \tag{1.1}$$

The 560-plet is a second rank tensor spinor $\Theta^{\mu\nu}$ where $\mu,\nu = 1\cdots 10$ are SO(10) indices. It is antisymmetric in the tensor indices, and traceless which leads to $16\times 45-160 = 560$

components. It has the $\mathsf{SU}(5)$ decomposition

$$\begin{aligned} \mathsf{560} = & \ \mathsf{1}(-5) + \overline{\mathsf{5}}(3) + \overline{\mathsf{10}}(-9) + \mathsf{10}_1(-1) + \mathsf{10}_2(-1) + \mathsf{24}(-5) + 40(-1) \\ & + \mathsf{45}(7) + \overline{\mathsf{45}}(3) + \overline{\mathsf{50}}(3) + \overline{\mathsf{70}}(3) + \mathsf{75}(-5) + \mathsf{175}(-1)\,. \end{aligned} \tag{1.2}$$

The $\mathsf{560}$ multiplet is remarkable in that it has features of $\mathsf{144}$ as well as of $\mathsf{126}$ in the following way. Thus from eq. (1.2) we see that in $\mathsf{SU}(5)$ decomposition it has three representations in it which carry non-vanishing $\mathsf{U}(1)$ quantum numbers, i.e., $\mathsf{1}(-5)$, $\mathsf{24}(-5)$, $\mathsf{75}(-5)$ and which can reduce the rank and break the GUT symmetry down to the Standard Model gauge group all at once just like the $\mathsf{144} + \overline{\mathsf{144}}$ does.

What makes it unique is that it also has $\overline{\mathsf{50}}(3)$ of $\mathsf{SU}(5)$ which as we know enters in the missing partner mechanism. Thus $\mathsf{560}$ contains one color triplet and four color anti-triplets, one up-type Higgs doublet, and three down-type doublets. Combined with $\overline{\mathsf{560}}$ one finds that $\mathsf{560} + \overline{\mathsf{560}}$ contains five pairs of Higgs color triplets/anti-triplets and four pairs of Higgs doublet pairs. If one allows a light Higgs sector consisting of $2{\times}\mathsf{10} + \mathsf{320}$ of Higgs fields, it brings in five pair of color triplets/anti-triplets and five pairs of light Higgs doublets. A mixing of the heavy and light sectors will then make all five color triplet pairs of the light sector and four Higgs doublets pairs of the light sector heavy leaving us with one pair of Higgs doublets from the light sector which would be the Higgs doublet pair of MSSM.

While some preliminary analysis of the model with $\mathsf{560} + \overline{\mathsf{560}}$ multiplet was given in reference [31] the details of this model have never been worked out. The main reason for it is the explicit computation of the interaction $\mathsf{560} \cdot \overline{\mathsf{560}} \cdot \mathsf{320}$ is difficult as it involves a special technique to handle spinor-tensor coupling. The technique to handle spinor-vector couplings involving $\mathsf{144}-$plet was developed in [31, 32] based on previous works [33–35] using oscillator technique [36, 37] and here we extend it to the coupling involving spinor-tensor. Thus one of the contributions of this work is that we carry out an explicit computation of these couplings and give a concrete analysis of the symmetry breaking chain. We identify the MSSM Higgs that emerges from the analysis as a linear combination of five Higgs doublet pairs from the light Higgs sector where two Higgs doublet pairs arise from $\mathsf{10}_1$ and $\mathsf{10}_2$ and three Higgs doublets arise from the $\mathsf{320}$-plet.

The outline of this paper is as follows: in section 2 we give a description of the model. In section 3 we give an analysis of breaking of $\mathsf{SO}(10)$ to $\mathsf{SU}(3)_\mathsf{C} \times \mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y}$ with a natural doublet-triplet splitting. An analysis of the Higgs doublet mass matrix is given in section 4. In section 5 we discuss the spontaneous breaking of $\mathsf{SO}(10)$ down to the standard model gauge group symmetry and generation of the light Higgs doublet. In section 6, phenomenological aspects of the model are discussed. Here it is shown that $b - t - \tau$ unification can be achieved at low $\tan\beta$ with $\tan\beta$ as low as 5–10. The analysis is carried out in $\tilde{g}$SUGRA framework where the radiative breaking of the electroweak symmetry is accomplished by a heavy gluino, which leads to a split sparticle mass spectrum and also a split Higgs boson mass spectrum. Thus the light spectrum lies in the few hundred GeV region and the heavy spectrum lies in the several TeV region consistent with the current experimental constraints on the Higgs boson mass, on the relic density of dark matter and on the muon anomaly. Further, it is shown that while the lightest sparticle (LSP) is the neutralino, the next to lightest particle (NLSP) is the light stau. An analysis of the production cross section for some for the sleptons and for the light weakinos is given and shown to be substantial indicating the possibility of detection of some of the light sparticles at 14 TeV high luminosity LHC (HL-LHC), and at 27 TeV high energy LHC (HE-LHC). Conclusions are given in section 7.

A more elaborate discussion of the mathematical details of the analysis are given in appendices A–G. Thus in appendix A we give an analysis of $560 + \overline{560}$ tensor-spinor in its $SU(5)$ oscillator modes. Further in this appendix we compute the normalization of kinetic energies of irreducible $SU(5)$ multiplets arising in the decomposition of $560 + \overline{560}$ multiplets. In appendix B, we give a computation of the couplings of the $SU(5)$ components fields 1, 24, 75 that appear in 560 and $\overline{560}$ multiplets. Such couplings determine the spontaneous breaking of $SO(10)$ to the Standard Model gauge group when they develop VEVs. In appendix C we exhibit extraction and normalization of all $SU(2)_L$ doublet fields appearing in the model. In appendix D we give a detailed discussion of how the symmetric ($\Lambda^{320_s}_{(\mu\nu)\lambda}$) and antisymmetric ($\Lambda^{320_a}_{[\mu\nu]\lambda}$) 320-dimensional three index tensors are constructed. In appendix E, we discuss the all the possible cubic couplings of the $320-$plet with the $560-$plet and their associated generational symmetries. In appendix F, we give extensive details for constructing normalized $70-$, $45-$ and $5-$plets with diagonal kinetic energy terms contained in $\Lambda^{320_s}_{(\mu\nu)\lambda}$ of $SO(10)$. The couplings of 320 and 10 to 560 and $\overline{560}$ multiplets are given in appendix G. These couplings play a central role in doublet-triplet splitting to produce a light Higgs that enters in electroweak symmetry breaking.

2 The model

The model makes use of a single $560 + \overline{560}$ multiplet, a single 320 multiplet and two 10 multiplets of $SO(10)$. The superpotential of the model is given by

$$\begin{aligned} W &= W_{\rm GUT} + W_{\rm DT} \\ W_{\rm GUT} &\sim 45 \cdot 45 + 560 \cdot \overline{560} \cdot 45 + 560 \cdot \overline{560}, \\ W_{\rm DT} &\sim 560 \cdot \overline{560} + 560 \cdot 560 \cdot 10_1 + \overline{560} \cdot \overline{560} \cdot (10_1 + 10_2) + 560 \cdot 560 \cdot 320 + \overline{560} \cdot \overline{560} \cdot 320, \end{aligned} \tag{2.1}$$

where $W_{\rm GUT}$ breaks the GUT symmetry down to the Standard Model gauge group and $W_{\rm DT}$ generates doublet-triplet splitting and leads to just one light Higgs doublet pair. The content of Higgs triplets/anti-triplets and of Higgs doublet pairs for $560 + \overline{560}$, for 320 and for $10_1 + 10_2$ is shown in table 1. In eq. (1.2) we exhibited the decomposition of 560 under $SU(5) \times U(1)$. The $SU(5) \times U(1)$ decomposition of 10-plets is simple, i.e., $10 = 5(2) + \overline{5}(-2)$ as noted earlier. For the 320 multiplet we have the decomposition

$$320 = 5(2) + \overline{5}(-2) + 40(-6) + \overline{40}(6) + 45(2) + \overline{45}(-2) + 70(2) + \overline{70}(-2). \tag{2.2}$$

Explicit tensorial structure of various terms in the superpotential will be discussed in sections below where we also give details of the computation of the couplings.

Heavy Fields	Light Fields	Pairs of D and T in Heavy Fields	Pairs of D and T in Light Fields	Residual Set of Light Modes
$560+\overline{560}$	$320+2\times 10$	4D+5T	(3D+3T)+ (2D+2T)	1D

Table 1. Exhibition of Higgs doublet pairs (D) consisting of up-type and down-type Higgs doublets, and Higgs triplet/anti-triplet (T) pairs in the $\mathsf{SO}(10)$ missing partner model discussed in this work.

Before proceeding further we exhibit the symmetric (s) and the antisymmetric (a) tensor products of 560 with itself.

$$\begin{aligned}(560\times 560)_s &= 10+126+\overline{126_1}+\overline{126_2}+210'+320+1728_1+1728_2+2970_1+2970_2\\ &\quad +3696'+4410+4950+\overline{4950}+6930'+10560+27720+36750+\overline{46800},\end{aligned}\tag{2.3}$$

$$\begin{aligned}(560\times 560)_a &= 120_1+120_2+320+1728_1+1728_2+2970+\overline{3696'_1}+\overline{3696'_2}+\overline{3696'_3}\\ &\quad +4312_1+4312_2+10560+34398+36750+\overline{48114}.\end{aligned}\tag{2.4}$$

Here we find that both the symmetric and the antisymmetric tensor products of 560 with itself contain a 320. Correspondingly there are two 320 multiplets which is symmetric in two of the three tensor indices with the totally symmetric part and the trace on two indices removed and the other which is antisymmetric in two of its indices with the totally antisymmetric part and the trace removed. In this analysis we consider the coupling of the 320_s to 560×560 and to $\overline{560}\times\overline{560}$. The irreducible tensor-spinor $|\Theta^{(560)}_{\mu\nu}\rangle$ can be obtained from the 720 $(=16\times 45)$ dimensional reducible tensor-spinor $|\Theta^{720}_{\mu\nu}\rangle$ by removing 160 $(=16\times 10)$ components, given by $\Gamma_\mu\,|\Theta^{(720)}_{\mu\nu}\rangle$, out of the 720 components of the unconstrained tensor-spinor. Thus we define the 560 multiplet so that it is antisymmetric in its two tensor indices and traceless given by

$$\text{Traceless:}\qquad \Gamma_\mu\,|\Theta^{\breve{s}\,(560)}_{\mu\nu}\rangle = 0,\tag{2.5a}$$

$$\text{Antisymmetric:}\qquad |\Theta^{\breve{s}\,(560)}_{\mu\nu}\rangle = -\,|\Theta^{\breve{s}\,(560)}_{\nu\mu}\rangle\,.\tag{2.5b}$$

Here $\breve{s}=1,\dots,16$ is the spinor index and the Greek letters $\mu,\nu,\dots=1,2,\dots,10$ are the $\mathsf{SO}(10)$ indices and Γ_μ satisfy the rank-10 Clifford algebra, that is, $\{\Gamma_\mu,\Gamma_\nu\}=2\delta_{\mu\nu}$. Further details of the construction of 560 and of $\overline{560}$ are discussed in appendix A where we also give an explicit decomposition of them in terms of irreducible representations of $\mathsf{SU}(5)\times\mathsf{U}(1)$.

Regarding 320 multiplet there are two: one of which is symmetric in two indices while the other is antisymmetric. We can construct them from the 1000 dimensional multiplet $T_{\mu\nu\lambda}$. Thus the antisymmetric one is given by

$$\Lambda^{(320_a)}_{[\mu\nu]\lambda}=\frac{1}{6}\Big[2(T_{\mu\nu\lambda}-T_{\nu\mu\lambda})+T_{\lambda\nu\mu}+T_{\mu\lambda\nu}-T_{\nu\lambda\mu}-T_{\lambda\mu\nu}\Big]-\frac{1}{18}\Big(\delta_{\nu\lambda}\delta_{\mu\beta}-\delta_{\mu\lambda}\delta_{\nu\beta}\Big)(T_{\beta\alpha\alpha}-T_{\alpha\beta\alpha})\tag{2.6}$$

while the symmetric 320-plet is given by

$$\begin{aligned}\Lambda^{(320_s)}_{(\mu\nu)\lambda}&=\frac{1}{6}\Big[2(T_{\mu\nu\lambda}+T_{\nu\mu\lambda})-(T_{\lambda\nu\mu}+T_{\mu\lambda\nu}+T_{\nu\lambda\mu}+T_{\lambda\mu\nu})\Big]\\ &\quad+\frac{1}{54}(\delta_{\nu\lambda}\delta_{\mu\beta}+\delta_{\mu\lambda}\delta_{\nu\beta}-2\delta_{\mu\nu}\delta_{\lambda\beta})(-T_{\beta\alpha\alpha}+2T_{\alpha\alpha\beta}-T_{\alpha\beta\alpha})\end{aligned}\tag{2.7}$$

The details of their construction are given in appendix D.

3 One step breaking of SO(10) to $\mathsf{SU(3)_C \times SU(2)_L \times U(1)_Y}$ and natural doublet-triplet splitting

The form of the superpotential responsible for a one-scale breaking of SO(10) symmetry is

$$W_{\rm GUT} \sim \ \mathsf{45} \cdot \mathsf{45} + \mathsf{560} \cdot \overline{\mathsf{560}} \cdot \mathsf{45} + \mathsf{560} \cdot \overline{\mathsf{560}}. \tag{3.1}$$

In the computation of these couplings we will use the method developed in [32–34] using oscillator techniques [36, 37]. Exhibiting its tensorial structure, the superpotential can be written as

$$W_{\rm GUT} = \frac{1}{2!} M_{45} \Phi^{(45)}_{\mu\nu} \Phi^{(45)}_{\mu\nu} + \lambda_{45} \left\langle \Theta^{(560)*}_{\mu\nu} | B | \overline{\Theta}^{(\overline{560})}_{\nu\sigma} \right\rangle \Phi^{(45)}_{\mu\sigma} + M_{560} \left\langle \Theta^{(560)*}_{\mu\nu} | B | \overline{\Theta}^{(\overline{560})}_{\mu\nu} \right\rangle , \tag{3.2}$$

where B is the SO(10) charge conjugation operator given by $B = \prod_{\mu={\rm odd}} \Gamma_\mu = -i \prod_{k=1}^{5} \left(b_k - b_k^\dagger \right)$. Integrating out $\Phi^{45}_{\mu\nu}$ in eq. (3.2), expanding out as in eqs. (B.2)–(B.4) and finally substituting eqs. (B.5)–(B.7) we get

$$\begin{aligned}
W_{\rm GUT} = \frac{\lambda_{45}^2}{4M_{45}} \Bigg[&- \left(\frac{7}{432}\right) \mathbf{S}_{75}^2 \overline{\mathbf{S}}_{75}^2 - \left(\frac{\sqrt{5}}{216}\right) \mathbf{S}_{75}^2 \overline{\mathbf{S}}_{75} \overline{\mathbf{S}}_{24} - \left(\frac{\sqrt{5}}{216}\right) \mathbf{S}_{75} \overline{\mathbf{S}}_{75}^2 \mathbf{S}_{24} \\
&- \left(\frac{349}{8640}\right) \mathbf{S}_{75} \overline{\mathbf{S}}_{75} \mathbf{S}_{24} \overline{\mathbf{S}}_{24} + \left(\frac{1}{180}\right) \mathbf{S}_{75} \overline{\mathbf{S}}_{75} \mathbf{S}_{24} \overline{\mathbf{S}}_{1} + \left(\frac{1}{180}\right) \mathbf{S}_{75} \overline{\mathbf{S}}_{75} \overline{\mathbf{S}}_{24} \mathbf{S}_{1} \\
&- \left(\frac{1}{90}\right) \mathbf{S}_{75} \overline{\mathbf{S}}_{75} \mathbf{S}_{1} \overline{\mathbf{S}}_{1} - \left(\frac{25}{3456}\right) \overline{\mathbf{S}}_{75}^2 \mathbf{S}_{24}^2 - \left(\frac{25}{3456}\right) \mathbf{S}_{75}^2 \overline{\mathbf{S}}_{24}^2 \\
&+ \left(\frac{\sqrt{5}}{288}\right) \mathbf{S}_{75} \overline{\mathbf{S}}_{24} \mathbf{S}_{24} \overline{\mathbf{S}}_{1} + \left(\frac{\sqrt{5}}{288}\right) \mathbf{S}_{75} \overline{\mathbf{S}}_{24}^2 \mathbf{S}_{1} + \left(\frac{\sqrt{5}}{288}\right) \overline{\mathbf{S}}_{75} \mathbf{S}_{24}^2 \overline{\mathbf{S}}_{1} \\
&+ \left(\frac{\sqrt{5}}{288}\right) \overline{\mathbf{S}}_{75} \mathbf{S}_{24} \overline{\mathbf{S}}_{24} \mathbf{S}_{1} - \left(\frac{49}{8640\sqrt{5}}\right) \mathbf{S}_{75} \overline{\mathbf{S}}_{24}^2 \mathbf{S}_{24} - \left(\frac{49}{8640\sqrt{5}}\right) \overline{\mathbf{S}}_{75} \mathbf{S}_{24}^2 \overline{\mathbf{S}}_{24} \\
&- \left(\frac{217}{17280}\right) \overline{\mathbf{S}}_{24}^2 \mathbf{S}_{24}^2 + \left(\frac{1}{1440}\right) \mathbf{S}_{24}^2 \overline{\mathbf{S}}_{24} \overline{\mathbf{S}}_{1} + \left(\frac{1}{1440}\right) \overline{\mathbf{S}}_{24}^2 \mathbf{S}_{24} \mathbf{S}_{1} \\
&- \left(\frac{1}{480}\right) \mathbf{S}_{24}^2 \overline{\mathbf{S}}_{1}^2 - \left(\frac{1}{480}\right) \overline{\mathbf{S}}_{24}^2 \mathbf{S}_{1}^2 + \left(\frac{1}{144}\right) \overline{\mathbf{S}}_{24} \mathbf{S}_{24} \mathbf{S}_{1} \overline{\mathbf{S}}_{1} - \left(\frac{1}{405}\right) \overline{\mathbf{S}}_{1}^2 \mathbf{S}_{1}^2 \Bigg] \\
&+ i M_{560} \left[\frac{1}{4} \overline{\mathbf{S}}_{75} \mathbf{S}_{75} + (1)\, \overline{\mathbf{S}}_{24} \mathbf{S}_{24} + (1)\, \overline{\mathbf{S}}_{1} \mathbf{S}_{1} \right] .
\end{aligned} \tag{3.3}$$

Here $\mathbf{S}$ and $\overline{\mathbf{S}}$ represent the $\mathsf{SU(3)_C \times SU(2)_L \times U(1)_Y}$ singlets contained in SU(5)'s $\mathsf{1}+\mathsf{24}+\mathsf{75}$ and $\overline{\mathsf{1}}+\overline{\mathsf{24}}+\overline{\mathsf{75}}$ multiplets. Further details of the analysis are given in the appendix B.

Vanishing of the F-terms requires that we look for solutions of eq. (3.3) for which

$$\frac{\partial W_{\rm GUT}}{\partial \mathbf{S}_{75}} = 0, \quad \frac{\partial W_{\rm GUT}}{\partial \overline{\mathbf{S}}_{75}} = 0, \quad \frac{\partial W_{\rm GUT}}{\partial \mathbf{S}_{24}} = 0, \quad \frac{\partial W_{\rm GUT}}{\partial \overline{\mathbf{S}}_{24}} = 0, \quad \frac{\partial W_{\rm GUT}}{\partial \mathbf{S}_{1}} = 0, \quad \frac{\partial W_{\rm GUT}}{\partial \overline{\mathbf{S}}_{1}} = 0, \tag{3.4}$$

are satisfied simultaneously. The solutions to the eqs. (3.4) are carried out numerically on Mathematica. The number of allowed solutions is quite large. We display solutions for the special case when the VEVs of $\mathsf{1}(-5)$ and $\overline{\mathsf{1}}(5)$, $\mathsf{24}(-5)$ and $\overline{\mathsf{24}}(5)$, $\mathsf{75}(-5)$ and $\overline{\mathsf{75}}(5)$ are equal. We choose to exhibit a total of 10 solutions (see table 2), two for each value of $M_{\rm eff}^2 \equiv \dfrac{M_{45} \cdot M_{560}}{\lambda_{45}^2}$ where we identify $M_{\rm eff}$ as the GUT scale, $M_{\rm GUT}$.

In this model the doublet-triplet splitting is automatic because of the missing partner constraint. The superpotential that triggers doublet-triplet splitting in symbolic form is given by

$$W_{\rm DT} \sim \mathsf{560}\cdot\overline{\mathsf{560}}+\mathsf{560}\cdot\mathsf{560}\cdot\mathsf{10}_1+\overline{\mathsf{560}}\cdot\overline{\mathsf{560}}\cdot(\mathsf{10}_1+\mathsf{10}_2)+\mathsf{560}\cdot\mathsf{560}\cdot\mathsf{320}+\overline{\mathsf{560}}\cdot\overline{\mathsf{560}}\cdot\mathsf{320}. \tag{3.5}$$

With full tensorial structure incorporated, eq. (3.5) takes the form

$$\begin{aligned} W_{\rm DT} = {} & M_{560}\ \langle\Theta^{(\mathsf{560})*}_{\mu\nu}|B|\overline{\Theta}^{(\overline{\mathsf{560}})}_{\mu\nu}\rangle + \alpha\ \langle\Theta^{(\mathsf{560})*}_{\mu\nu}|B\Gamma_\alpha|\Theta^{(\mathsf{560})}_{\mu\nu}\rangle\,\Omega^{(\mathsf{10}_1)}_\alpha \\ & +\sum_{r=1}^{2}\overline{\alpha}_r\ \langle\overline{\Theta}^{(\overline{\mathsf{560}})*}_{\mu\nu}|B\Gamma_\alpha|\overline{\Theta}^{(\overline{\mathsf{560}})}_{\mu\nu}\rangle\,\Omega^{(\mathsf{10_r})}_\alpha \\ & +\beta\ \langle\Theta^{(\mathsf{560})*}_{\mu\sigma}|B\Gamma_\lambda|\Theta^{(\mathsf{560})}_{\nu\sigma}\rangle\,\Lambda^{(\mathsf{320_s})}_{(\mu\nu)\lambda}+\overline{\beta}\ \langle\overline{\Theta}^{(\overline{\mathsf{560}})*}_{\mu\sigma}|B\Gamma_\lambda|\overline{\Theta}^{(\overline{\mathsf{560}})}_{\nu\sigma}\rangle\,\Lambda^{(\mathsf{320_s})}_{(\mu\nu)\lambda}. \end{aligned} \tag{3.6}$$

Further details of the analysis are given in appendix G.

4 The Higgs doublet mass matrix

As shown in table 1 the Higgs boson doublets arise from the heavy fields $\mathsf{560}+\overline{\mathsf{560}}$ and the light fields which consists of two $\mathsf{10}-$plets and a $\mathsf{320}-$plet of $\mathsf{SO(10)}$. We list them below.

$$\{\mathcal{D}^i\} = \left({}^{(5_{10_1})}\mathbf{D}^a, {}^{(5_{10_2})}\mathbf{D}^a, {}^{(5_{320})}\mathbf{D}^a, {}^{(5_{\overline{560}})}\mathbf{D}^a {}^{(45_{320})}\mathbf{D}^a, {}^{(45_{560})}\mathbf{D}^a, {}^{(45_{\overline{560}})}\mathbf{D}^a, {}^{(70_{320})}\mathbf{D}^a, {}^{(70_{\overline{560}})}\mathbf{D}^a\right). \tag{4.1}$$

Similarly we have the anti-doublets given by

$$\{\overline{\mathcal{D}}_j\} = \left({}^{(\bar{5}_{10_1})}\mathbf{D}_a, {}^{(\bar{5}_{10_2})}\mathbf{D}_a, {}^{(\bar{5}_{320})}\mathbf{D}_a, {}^{(\bar{5}_{560})}\mathbf{D}_a {}^{(\overline{45}_{320})}\mathbf{D}_a, {}^{(\overline{45}_{\overline{560}})}\mathbf{D}_a, {}^{(\overline{45}_{560})}\mathbf{D}_a, {}^{(\overline{70}_{320})}\mathbf{D}_a, {}^{(\overline{70}_{560})}\mathbf{D}_a\right). \tag{4.2}$$

The above leads to a 9×9 dimensional Higgs doublet mass matrix. The mass matrix is constructed from the superpotential of eq. (3.6) after spontaneous breaking and one has the following result:

$$M_{\rm doublet} = \begin{array}{c|ccccccccc} & {}^{(\bar{5}_{10_1})}\mathbf{D}_a & {}^{(\bar{5}_{10_2})}\mathbf{D}_a & {}^{(\bar{5}_{320})}\mathbf{D}_a & {}^{(\bar{5}_{560})}\mathbf{D}_a & {}^{(\overline{45}_{320})}\mathbf{D}_a & {}^{(\overline{45}_{\overline{560}})}\mathbf{D}_a & {}^{(\overline{45}_{560})}\mathbf{D}_a & {}^{(\overline{70}_{320})}\mathbf{D}_a & {}^{(\overline{70}_{560})}\mathbf{D}_a \\ \hline {}^{(5_{10_1})}\mathbf{D}^a & 0 & 0 & 0 & d_3 & 0 & \bar{d}_2 & d_4 & 0 & d_5 \\ {}^{(5_{10_2})}\mathbf{D}^a & 0 & 0 & 0 & 0 & 0 & \frac{\bar{d}_2\bar{\alpha}_2}{\bar{\alpha}_1} & 0 & 0 & 0 \\ {}^{(5_{320})}\mathbf{D}^a & 0 & 0 & 0 & d_7 & 0 & \bar{d}_6 & d_8 & 0 & d_{15} \\ {}^{(5_{\overline{560}})}\mathbf{D}^a & \bar{d}_3 & \frac{\bar{d}_3\bar{\alpha}_2}{\bar{\alpha}_1} & \bar{d}_7 & d_1 & \bar{d}_{13} & 0 & 0 & \bar{d}_{16} & 0 \\ {}^{(45_{320})}\mathbf{D}^a & 0 & 0 & 0 & d_{13} & 0 & \bar{d}_{14} & d_{12} & 0 & 0 \\ {}^{(45_{560})}\mathbf{D}^a & d_2 & 0 & d_6 & 0 & d_{14} & -\frac{d_1}{2} & 0 & d_{11} & 0 \\ {}^{(45_{\overline{560}})}\mathbf{D}^a & \bar{d}_4 & \frac{\bar{d}_4\bar{\alpha}_2}{\bar{\alpha}_1} & \bar{d}_8 & 0 & \bar{d}_{12} & 0 & \frac{d_1}{2} & \bar{d}_9 & 0 \\ {}^{(70_{320})}\mathbf{D}^a & 0 & 0 & 0 & d_{16} & 0 & \bar{d}_{11} & d_9 & 0 & d_{10} \\ {}^{(70_{\overline{560}})}\mathbf{D}^a & \bar{d}_5 & \frac{d_5\bar{\alpha}_2}{\bar{\alpha}_1} & \bar{d}_{15} & 0 & 0 & 0 & 0 & \bar{d}_{10} & \frac{d_1}{2} \end{array}, \tag{4.3}$$

where d_i and $\bar{d}_j$ are defined as follows

$$
\begin{aligned}
d_1 &= iM_{560}; & d_2 &= \frac{-i\alpha}{2\sqrt{6}}\left(\sqrt{5}\mathcal{V}_{24}+4\mathcal{V}_{75}\right),\\
d_3 &= i\alpha\sqrt{\frac{2}{10815}}\,(97\mathcal{V}_1+6\mathcal{V}_{24}); & d_4 &= \frac{i\alpha}{5\sqrt{87}}\left(25\mathcal{V}_{24}-\sqrt{5}\mathcal{V}_{75}\right),\\
d_5 &= -\frac{1}{4}\sqrt{\frac{15}{2}}i\alpha\mathcal{V}_{24}; & d_6 &= \frac{i\beta}{144\sqrt{2}}\left(23\sqrt{5}\mathcal{V}_{24}-16\mathcal{V}_{75}\right),\\
d_7 &= \frac{i\beta}{504\sqrt{7210}}\,(13480\mathcal{V}_1+7761\mathcal{V}_{24}); & d_8 &= \frac{i\beta}{1008\sqrt{29}}\left(2695\mathcal{V}_{24}+212\sqrt{5}\mathcal{V}_{75}\right),\\
d_9 &= \frac{i\beta}{120\sqrt{174}}\left(65\mathcal{V}_{24}+184\sqrt{5}\mathcal{V}_{75}\right); & d_{10} &= \frac{i\beta}{40\sqrt{3}}\left(-8\sqrt{5}\mathcal{V}_1-\sqrt{5}\mathcal{V}_{24}+20\mathcal{V}_{75}\right),\\
d_{11} &= \frac{i\beta}{48\sqrt{3}}\left(-5\sqrt{5}\mathcal{V}_{24}+16\mathcal{V}_{75}\right); & d_{12} &= \frac{i\beta}{60\sqrt{29}}\left(196\mathcal{V}_1-129\mathcal{V}_{24}+2\sqrt{5}\mathcal{V}_{75}\right),\\
d_{13} &= \frac{i\beta}{24\sqrt{1442}}\left(217\sqrt{5}\mathcal{V}_{24}-8\mathcal{V}_{75}\right); & d_{14} &= \frac{i\beta}{48\sqrt{10}}\,(-16\mathcal{V}_1+33\mathcal{V}_{24}),\\
d_{15} &= \frac{37i\beta}{168}\sqrt{\frac{5}{2}}\mathcal{V}_{24}; & d_{16} &= \frac{37i\beta}{8}\sqrt{\frac{5}{2163}}\mathcal{V}_{24},\\
\bar{d}_2 &= \frac{-i\bar{\alpha}}{2\sqrt{6}}\left(\sqrt{5}\overline{\mathcal{V}}_{24}+4\overline{\mathcal{V}}_{75}\right); & \bar{d}_3 &= i\bar{\alpha}\sqrt{\frac{2}{10815}}\left(97\overline{\mathcal{V}}_1+6\overline{\mathcal{V}}_{24}\right),\\
\bar{d}_4 &= \frac{i\bar{\alpha}}{5\sqrt{87}}\left(25\overline{\mathcal{V}}_{24}-\sqrt{5}\overline{\mathcal{V}}_{75}\right); & \bar{d}_5 &= -\frac{1}{4}\sqrt{\frac{15}{2}}i\bar{\alpha}\overline{\mathcal{V}}_{24},\\
\bar{d}_6 &= \frac{i\bar{\beta}}{144\sqrt{2}}\left(23\sqrt{5}\overline{\mathcal{V}}_{24}-16\overline{\mathcal{V}}_{75}\right); & \bar{d}_7 &= \frac{i\bar{\beta}}{504\sqrt{7210}}(13480\overline{\mathcal{V}}_1+7761\overline{\mathcal{V}}_{24}),\\
\bar{d}_8 &= \frac{i\bar{\beta}}{1008\sqrt{29}}\left(2695\overline{\mathcal{V}}_{24}+212\sqrt{5}\overline{\mathcal{V}}_{75}\right); & \bar{d}_9 &= \frac{i\bar{\beta}}{120\sqrt{174}}\left(65\overline{\mathcal{V}}_{24}+84\sqrt{5}\overline{\mathcal{V}}_{75}\right),\\
\bar{d}_{10} &= \frac{i\bar{\beta}}{40\sqrt{3}}\left(-8\sqrt{5}\overline{\mathcal{V}}_1-\sqrt{5}\overline{\mathcal{V}}_{24}+20\overline{\mathcal{V}}_{75}\right); & \bar{d}_{11} &= \frac{i\bar{\beta}}{48\sqrt{3}}\left(-5\sqrt{5}\overline{\mathcal{V}}_{24}+16\overline{\mathcal{V}}_{75}\right),\\
\bar{d}_{12} &= \frac{i\bar{\beta}}{60\sqrt{29}}\left(196\overline{\mathcal{V}}_1-129\overline{\mathcal{V}}_{24}+2\sqrt{5}\overline{\mathcal{V}}_{75}\right); & \bar{d}_{13} &= \frac{i\bar{\beta}}{24\sqrt{1442}}\left(217\sqrt{5}\overline{\mathcal{V}}_{24}-8\overline{\mathcal{V}}_{75}\right),\\
\bar{d}_{14} &= \frac{i\bar{\beta}}{48\sqrt{10}}(-16\overline{\mathcal{V}}_1+33\overline{\mathcal{V}}_{24}); & \bar{d}_{15} &= \frac{37i\bar{\beta}}{168}\sqrt{\frac{5}{2}}\overline{\mathcal{V}}_{24},\\
\bar{d}_{16} &= \frac{37i\bar{\beta}}{8}\sqrt{\frac{5}{2163}}\overline{\mathcal{V}}_{24}. & & \qquad (4.4)
\end{aligned}
$$

The Higgs doublet mass matrix is diagonalized by two unitary matrices U and V such that

$$U_d^{\dagger}M_{\text{doublet}}V_d = M_{\text{doublet}}^{\text{diag}} = (m_{d_1},\ m_{d_2},\cdots,\ m_{d_8},0), \qquad (4.5)$$

whose relevant elements are displayed in eq. (4.6),

$$
\begin{pmatrix}
{}^{(\bar{5}_{10_1})}\mathbf{D}_a \\
{}^{(\bar{5}_{10_2})}\mathbf{D}_a \\
{}^{(\bar{5}_{320})}\mathbf{D}_a \\
{}^{(\bar{5}_{\overline{560}})}\mathbf{D}_a \\
{}^{(\overline{45}_{320})}\mathbf{D}_a \\
{}^{(\overline{45}_{\overline{560}})}\mathbf{D}_a \\
{}^{(\overline{45}_{560})}\mathbf{D}_a \\
{}^{(\overline{70}_{320})}\mathbf{D}_a \\
{}^{(\overline{70}_{560})}\mathbf{D}_a
\end{pmatrix}
=
\begin{pmatrix}
\cdots & \cdots & V_{d_{19}} \\
\cdots & \cdots & V_{d_{29}} \\
\cdots & \cdots & V_{d_{39}} \\
\cdots & \cdots & 0 \\
\cdots & \cdots & V_{d_{59}} \\
\cdots & \cdots & 0 \\
\cdots & \cdots & 0 \\
\cdots & \cdots & V_{d_{89}} \\
\cdots & \cdots & 0
\end{pmatrix}
\begin{pmatrix}
{}^{1}\mathbf{D}'_a \\
{}^{2}\mathbf{D}'_a \\
{}^{3}\mathbf{D}'_a \\
{}^{4}\mathbf{D}'_a \\
{}^{5}\mathbf{D}'_a \\
{}^{6}\mathbf{D}'_a \\
{}^{7}\mathbf{D}'_a \\
{}^{8}\mathbf{D}'_a \\
\mathbf{H_d}_a
\end{pmatrix};
\quad
\begin{pmatrix}
{}^{(5_{10_1})}\mathbf{D}^a \\
{}^{(5_{10_2})}\mathbf{D}^a \\
{}^{(5_{320})}\mathbf{D}^a \\
{}^{(5_{\overline{560}})}\mathbf{D}^a \\
{}^{(45_{320})}\mathbf{D}^a \\
{}^{(45_{560})}\mathbf{D}^a \\
{}^{(45_{\overline{560}})}\mathbf{D}^a \\
{}^{(70_{320})}\mathbf{D}^a \\
{}^{(70_{\overline{560}})}\mathbf{D}^a
\end{pmatrix}
=
\begin{pmatrix}
\cdots & \cdots & U_{d_{19}} \\
\cdots & \cdots & U_{d_{29}} \\
\cdots & \cdots & U_{d_{39}} \\
\cdots & \cdots & 0 \\
\cdots & \cdots & U_{d_{59}} \\
\cdots & \cdots & 0 \\
\cdots & \cdots & 0 \\
\cdots & \cdots & U_{d_{89}} \\
\cdots & \cdots & 0
\end{pmatrix}
\begin{pmatrix}
{}^{1}\mathbf{D}'^a \\
{}^{2}\mathbf{D}'^a \\
{}^{3}\mathbf{D}'^a \\
{}^{4}\mathbf{D}'^a \\
{}^{5}\mathbf{D}'^a \\
{}^{6}\mathbf{D}'^a \\
{}^{7}\mathbf{D}'^a \\
{}^{8}\mathbf{D}'^a \\
\mathbf{H_u}^a
\end{pmatrix},
\tag{4.6}
$$

where $\mathbf{D}$'s and $\mathbf{D}'$'s represent the normalized kinetic energy basis and normalized kinetic and mass eigenbasis, respectively of the doublet mass matrix of eq. (4.3). The pair of doublets $(\mathbf{H_d}_a, \mathbf{H_u}^a)$ are identified to be light and are the normalized electroweak Higgs doublets of the minimal supersymmetric standard model (MSSM). Numerical values of the non-zero matrix elements of U and V are displayed in tables 3 and 4 for benchmarks of table 2.

The doublet mass matrix has a zero mode which gives us the electroweak doublet pair of MSSM, i.e., $\mathbf{H_u}^a$ which couples to the up-quarks and $\mathbf{H_d}_a$ which couples to the down quarks and the leptons. The electroweak doublets do not have any components in the doublets arising from $\mathsf{560}$ and $\overline{\mathsf{560}}$ since they are superheavy and are linear combinations only of the doublets arising from $\mathsf{10}_1$ and $\mathsf{10}_2$ of Higgs and the doublets arising from the $\mathsf{320}-$plet of Higgs. Thus one may write $\mathbf{H_u}^a$ and $\mathbf{H_d}_a$ as the following combination of fields

$$
\mathbf{H_u}^a = \sum_{k=1}^{5} c_{uk} D^a_k, \quad \mathbf{H_d}_a = \sum_{k=1}^{5} c_{dk} D_{ak} \tag{4.7}
$$

where

$$
\left\{D^a_{k=1,\ldots,5}\right\} = \left({}^{(5_{10_1})}\mathbf{D}^a,\ {}^{(5_{10_2})}\mathbf{D}^a,\ {}^{(5_{320})}\mathbf{D}^a,\ {}^{(45_{320})}\mathbf{D}^a,\ {}^{(70_{320})}\mathbf{D}^a\right). \tag{4.8}
$$

Similarly we have the anti-doublets which we display as

$$
\{D_{ak=1,\ldots,5}\} = \left({}^{(\bar{5}_{10_1})}\mathbf{D}_a,\ {}^{(\bar{5}_{10_2})}\mathbf{D}_a,\ {}^{(\bar{5}_{320})}\mathbf{D}_a,\ {}^{(\overline{45}_{320})}\mathbf{D}_a,\ {}^{(\overline{70}_{320})}\mathbf{D}_a\right), \tag{4.9}
$$

where c_{uk} and c_{dk} are determined by diagonalizing Higgs doublet mass matrix of eq. (4.3). The $\mathsf{320}$ has no cubic couplings with quarks and leptons and thus only the components of the two Higgs doublets arising from $\mathsf{10}_1$ and $\mathsf{10}_2$ (and not all five components arising from eqs. (4.7) and (4.8)) enter in the couplings of the Higgs bosons to quarks and leptons.

5 Numerical analysis of breaking of $\mathsf{SO(10)}$ to $\mathsf{SU(3)_C} \times \mathsf{SU(2)_L} \times \mathsf{U(1)_Y}$ and generation of light Higgs doublets

We discuss now the numerical analysis of the $\mathsf{560} + \overline{\mathsf{560}} + \mathsf{320}+2\times\mathsf{10}$ model to break the $\mathsf{SO(10)}$ symmetry to the Standard Model gauge group, and analyze the Higgs doublet structure

$M_{\rm eff}^2 \equiv \dfrac{M_{45} \cdot M_{560}}{\lambda_{45}^2}$ (GeV^2)	$\mathcal{V}_{75}$ (GeV)	$\mathcal{V}_{24}$ (GeV)	$\mathcal{V}_1$ (GeV)
	$\mathcal{V}_{75} = \overline{\mathcal{V}}_{75}$, $\mathcal{V}_{24} = \overline{\mathcal{V}}_{24}$, $\mathcal{V}_1 = \overline{\mathcal{V}}_1$		
10^{30}	$(-1.082 + \imath 2.927) \times 10^{16}$	$(-2.844 - \imath 6.446) \times 10^{15}$	$(-4.716 - \imath 2.297) \times 10^{16}$
	$(7.762 + \imath 8.288) \times 10^{15}$	$(-7.933 + \imath 6.054) \times 10^{15}$	$(1.156 + \imath 0.980) \times 10^{16}$
10^{31}	$(7.205 + \imath 7.205) \times 10^{14}$	$(-3.356 - \imath 3.356) \times 10^{16}$	$(6.813 + \imath 6.813) \times 10^{16}$
	$(-1.661 - \imath 2.608) \times 10^{16}$	$(2.207 - \imath 0.100) \times 10^{16}$	$(3.578 - \imath 5.238) \times 10^{15}$
10^{32}	$(-0.588 - \imath 1.223) \times 10^{17}$	$(1.342 - \imath 0.251) \times 10^{17}$	$(1.731 + \imath 1.577) \times 10^{17}$
	$(-3.691 - \imath 3.691) \times 10^{16}$	$(-4.226 - \imath 4.226) \times 10^{15}$	$(2.287 + \imath 2.287) \times 10^{14}$
10^{33}	$(-9.258 + \imath 3.422) \times 10^{17}$	$(2.038 + \imath 0.899) \times 10^{17}$	$(0.726 + \imath 1.491) \times 10^{18}$
	$(2.621 + \imath 2.455) \times 10^{17}$	$(1.915 - \imath 2.509) \times 10^{17}$	$(3.098 - \imath 3.655) \times 10^{17}$
10^{34}	$(-8.248 - \imath 5.251) \times 10^{17}$	$(-0.318 + \imath 6.978) \times 10^{17}$	$(-1.657 + \imath 1.132) \times 10^{17}$
	$(-0.588 - \imath 1.223) \times 10^{18}$	$(1.342 - \imath 0.251) \times 10^{18}$	$(1.731 + \imath 1.577) \times 10^{18}$

Table 2. Numerical estimates of the VEVs of the singlet, the 24−plet and the 75−plet fields in $560 + \overline{560}$ multiplets in the spontaneous breaking of the SO(10) gauge symmetry to SU(5) × U(1) at the GUT scale.

to exhibit explicitly a pair of light Higgs doublets that enter in the electroweak symmetry breaking and would generate quark and lepton masses. First we will numerically solve the spontaneous symmetry breaking equations in section 3 and determine the allowed values of VEVs $\mathcal{V}_1, \mathcal{V}_{24}, \mathcal{V}_{75}$. The mass scale that enters in the determination of these VEVs is $M_{\rm eff}$ where $M_{\rm eff}^2 = M_{45} \cdot M_{560}/\lambda_{45}^2$. To determine the VEVs we solve the spontaneous symmetry breaking equations eqs. (3.4). We list in table 2, the computations of $\mathcal{V}_1, \mathcal{V}_{24}, \mathcal{V}_{75}$ for a range of $M_{\rm eff}$ from 10^{15} GeV to 10^{17} GeV. We note that all of the VEVs, i.e., $\mathcal{V}_1$, $\mathcal{V}_{24}$, and $\mathcal{V}_{75}$ are comparable in size and thus enter essentially with equal strength in breaking the gauge group SO(10) down to the Standard Model gauge group. The analysis of table 2 is then used to numerically compute the Higgs doublet mass matrix of eq. (4.3) where the elements of this mass matrix consisting of $d_1 \cdots d_{16}$, $\bar{d}_6, \cdots, \bar{d}_{16}$ given by eq. (4.4) are determined in terms of the heavy VEVs $\mathcal{V}_1$, $\mathcal{V}_{24}$, and $\mathcal{V}_{75}$. Diagonalization of the mass matrix of eq. (4.3) by the biunitary transformation of eq. (4.5) allows us to identify the massless modes, i.e., the electroweak doublet. The exact composition of the electroweak doublet is given by the transformations of eq. (4.6) which connect the primitive field doublets labeled by $\mathbf{D}_a, \mathbf{D}^a$ to the ones in mass diagonal basis $\mathbf{D}'_a, \mathbf{D}'^a$. The composition of the massless doublets is given in terms of the diagonalizing matrices U and V that appear in eq. (4.6). The numerical composition of the massless up-Higgs is given in table 3 for a range of $M_{\rm eff}$ values as given in table 2. Similarly the numerical composition of the massless down-Higgs is given in table 4 for same range of $M_{\rm eff}$ values. The analysis above allows a computation of the quark and lepton masses using the VEVs of the electroweak doublets.

6 Yukawa unification, light sparticles, and SUSY implications at LHC

In this section, we discuss several phenomenological aspects of the model within supergravity grand unification [38–40]. These include Yukawa unification for the third generation of

$M_{560} = 1.0 \times 10^{18}$ GeV, $\alpha = \overline{\alpha}_1 = \overline{\alpha}_2 = 1.0$, $\beta = \overline{\beta} = 1.0$

$\mathcal{V}_{75}$ (GeV)	$\mathcal{V}_{24}$ (GeV)	$\mathcal{V}_1$ (GeV)	$U_{d_{19}}$	$U_{d_{29}}$	$U_{d_{39}}$	$U_{d_{59}}$	$U_{d_{89}}$
$(-1.082 + \imath 2.927) \times 10^{16}$	$(-2.844 - \imath 6.446) \times 10^{15}$	$(-4.716 - \imath 2.297) \times 10^{16}$	$0.230 + \imath 0.000$	$-0.169 + \imath 0.074$	$-0.926 - \imath 0.058$	$0.059 + \imath 0.097$	$-0.196 + \imath 0.020$
$(7.762 + \imath 8.288) \times 10^{15}$	$(-7.933 + \imath 6.054) \times 10^{15}$	$(1.156 + \imath 0.980) \times 10^{16}$	$-0.019 + \imath 0.000$	$0.289 + \imath 0.239$	$-0.052 + \imath 0.147$	$-0.232 + \imath 0.170$	$0.846 - \imath 0.193$
$(7.205 + \imath 7.205) \times 10^{14}$	$(-3.356 - \imath 3.356) \times 10^{16}$	$(6.813 + \imath 6.813) \times 10^{16}$	$0.112 + \imath 0.000$	$-0.672 + \imath 0.000$	$-0.504 + \imath 0.000$	$-0.105 + \imath 0.000$	$0.520 + \imath 0.000$
$(-1.661 - \imath 2.608) \times 10^{16}$	$(2.207 - \imath 0.100) \times 10^{16}$	$(3.578 - \imath 5.238) \times 10^{15}$	$-0.196 + \imath 0.000$	$0.189 + \imath 0.230$	$0.370 - \imath 0.314$	$-0.361 + \imath 0.073$	$0.648 + \imath 0.286$
$(-0.588 - \imath 1.223) \times 10^{17}$	$(1.342 - \imath 0.251) \times 10^{17}$	$(1.731 + \imath 1.577) \times 10^{17}$	$0.235 + \imath 0.000$	$-0.084 + \imath 0.108$	$-0.715 + \imath 0.257$	$-0.003 + \imath 0.145$	$-0.390 - \imath 0.418$
$(-3.691 - \imath 3.691) \times 10^{16}$	$(-4.226 - \imath 4.226) \times 10^{15}$	$(2.287 + \imath 2.287) \times 10^{14}$	$-0.630 + \imath 0.000$	$0.552 + \imath 0.000$	$0.489 + \imath 0.000$	$-0.045 + \imath 0.000$	$-0.240 + \imath 0.000$
$(-9.258 + \imath 3.422) \times 10^{17}$	$(2.038 + \imath 0.899) \times 10^{17}$	$(0.726 + \imath 1.491) \times 10^{18}$	$-0.230 + \imath 0.000$	$0.169 + \imath 0.074$	$0.926 - \imath 0.058$	$-0.059 + \imath 0.097$	$0.196 + \imath 0.020$
$(2.621 + \imath 2.455) \times 10^{17}$	$(1.915 - \imath 2.509) \times 10^{17}$	$(3.098 + \imath 3.655) \times 10^{17}$	$-0.019 + \imath 0.000$	$0.289 - \imath 0.239$	$-0.052 - \imath 0.147$	$-0.232 - \imath 0.170$	$0.846 + \imath 0.193$
$(-8.248 - \imath 5.252) \times 10^{17}$	$(-0.318 + \imath 6.978) \times 10^{17}$	$(-1.657 + \imath 1.132) \times 10^{17}$	$0.196 + \imath 0.000$	$-0.189 + \imath 0.230$	$-0.370 - \imath 0.314$	$0.361 + \imath 0.073$	$-0.648 + \imath 0.286$
$(-0.588 - \imath 1.223) \times 10^{18}$	$(1.342 - \imath 0.251) \times 10^{18}$	$(1.731 + \imath 1.577) \times 10^{18}$	$-0.235 + \imath 0.000$	$0.084 - \imath 0.108$	$0.715 - \imath 0.257$	$0.003 - \imath 0.145$	$0.390 + \imath 0.418$

Table 3. A numerical estimate of the elements of the *up* Higgs zero mode eigenvector using the analysis of table 2 and the Higgs doublet mass matrix, M_{doublet}, displayed in eq. (4.3) and U and V defined in eq. (4.5).

$M_{560} = 1.0 \times 10^{18}$ GeV, $\alpha = \overline{\alpha}_1 = \overline{\alpha}_2 = 1.0$, $\beta = \overline{\beta} = 1.0$

$\mathcal{V}_{75}$ (GeV)	$\mathcal{V}_{24}$ (GeV)	$\mathcal{V}_1$ (GeV)	$V_{d_{19}}$	$V_{d_{29}}$	$V_{d_{39}}$	$V_{d_{59}}$	$V_{d_{89}}$
$(-1.082 + \imath 2.927) \times 10^{16}$	$(-2.844 - \imath 6.446) \times 10^{15}$	$(-4.716 - \imath 2.297) \times 10^{16}$	$0.086 + \imath 0.049$	$-0.044 + \imath 0.183$	$-0.223 - \imath 0.923$	$0.109 + \imath 0.043$	$-0.014 - \imath 0.201$
$(7.762 + \imath 8.288) \times 10^{15}$	$(-7.933 + \imath 6.054) \times 10^{15}$	$(1.156 + \imath 0.980) \times 10^{16}$	$-0.296 + \imath 0.166$	$0.313 - \imath 0.162$	$0.018 + \imath 0.145$	$0.180 + \imath 0.202$	$-0.740 - \imath 0.345$
$(7.205 + \imath 7.205) \times 10^{14}$	$(-3.356 - \imath 3.356) \times 10^{16}$	$(6.813 + \imath 6.813) \times 10^{16}$	$0.408 - \imath 0.272$	$-0.490 + \imath 0.327$	$0.368 - \imath 0.246$	$0.076 - \imath 0.051$	$-0.379 + \imath 0.253$
$(-1.661 - \imath 2.608) \times 10^{16}$	$(2.207 - \imath 0.100) \times 10^{16}$	$(3.578 - \imath 5.238) \times 10^{15}$	$-0.115 - \imath 0.197$	$-0.057 + \imath 0.290$	$0.472 + \imath 0.100$	$-0.350 + \imath 0.106$	$0.432 - \imath 0.554$
$(-0.588 - \imath 1.223) \times 10^{17}$	$(1.342 - \imath 0.251) \times 10^{17}$	$(1.731 + \imath 1.577) \times 10^{17}$	$-0.022 + \imath 0.186$	$-0.138 - \imath 0.011$	$0.680 - \imath 0.358$	$0.110 + \imath 0.096$	$-0.046 - \imath 0.576$
$(-3.691 - \imath 3.691) \times 10^{16}$	$(-4.226 - \imath 4.226) \times 10^{15}$	$(2.287 + \imath 2.287) \times 10^{14}$	$0.036 - \imath 0.094$	$0.250 - \imath 0.661$	$-0.222 + \imath 0.586$	$0.020 - \imath 0.054$	$0.109 - \imath 0.288$
$(-9.258 + \imath 3.422) \times 10^{17}$	$(2.038 + \imath 0.899) \times 10^{17}$	$(0.726 + \imath 1.491) \times 10^{18}$	$-0.075 - \imath 0.064$	$-0.157 + \imath 0.103$	$0.944 - \imath 0.098$	$-0.076 - \imath 0.088$	$0.194 - \imath 0.053$
$(2.621 + \imath 2.455) \times 10^{17}$	$(1.915 - \imath 2.509) \times 10^{17}$	$(3.098 - \imath 3.655) \times 10^{17}$	$-0.088 - \imath 0.328$	$0.103 - \imath 0.338$	$0.117 - \imath 0.088$	$0.270 - \imath 0.012$	$-0.764 - \imath 0.288$
$(-8.248 - \imath 5.252) \times 10^{17}$	$(-0.318 + \imath 6.978) \times 10^{17}$	$(-1.657 + \imath 1.132) \times 10^{17}$	$-0.047 + \imath 0.224$	$-0.146 - \imath 0.257$	$0.415 - \imath 0.245$	$-0.365 + \imath 0.011$	$0.586 + \imath 0.389$
$(-0.588 - \imath 1.223) \times 10^{18}$	$(1.342 - \imath 0.251) \times 10^{18}$	$(1.731 + \imath 1.577) \times 10^{18}$	$0.185 - \imath 0.029$	$0.027 + \imath 0.136$	$-0.528 - \imath 0.558$	$0.063 - \imath 0.132$	$-0.542 + \imath 0.200$

Table 4. A numerical estimate of the elements of the *down* Higgs zero mode eigenvector using the analysis of table 2 and the Higgs doublet mass matrix, M_{doublet}, displayed in eq. (4.3) and U and V defined in eq. (4.5).

quarks and leptons, the muon anomaly, a split sparticle spectrum with a compressed low mass sparticle spectrum lying in the few hundred GeV region and a high mass spectrum with masses ranging over several TeV. The compressed low mass spectrum looks promising for SUSY discovery at the LHC. Regarding $b - t - \tau$ unification, in typical $\mathsf{SO(10)}$ models such a unification comes about for a large $\tan\beta$, as large as $\tan\beta = 50$ [41]. However, for the case of $\mathsf{SO(10)}$ under consideration here, one finds that a splitting of the $b - t$ Yukawa couplings occurs at the GUT scale which leads to a $b - t - \tau$ unification with a low $\tan\beta$ which can be as low as $\tan\beta = 5 - 10$. We give now some further details of the Yukawa unification analysis. We note that since there is no cubic interaction of matter fermions with the $\mathsf{320}$ and only $\mathsf{10}_1, \mathsf{10}_2$ enters in the Yukawa couplings, so the cubic superpotential of fermions with the Higgs fields takes the form

$$W_3 = \sum_{r=1}^{2} f^{10_r} \left\langle \Psi^*_{(+)} | B\Gamma_\mu | \Psi_{(+)} \right\rangle^r \Omega_\mu \tag{6.1}$$

Thus the third generation Yukawas are given by

$$h_\tau^0 = i2\sqrt{2} \sum_{r=1}^{2} f^{10_r} V_{d_{r9}}, \quad h_b^0 = -i2\sqrt{2} \sum_{r=1}^{2} f^{10_r} V_{d_{r9}}, \quad h_t^0 = -i2\sqrt{2} \sum_{r=1}^{2} f^{10_r} U_{d_{r9}}. \tag{6.2}$$

Gaugino masses from dim 5 operators

SU(5) rep	n_1^r	n_2^r	n_3^r
1	1	1	1
24	-1	-3	2
75	-5	3	1
200	10	2	1

Table 5. An exhibition of the contribution of the irreducible representations $\mathsf{1}, \mathsf{24}, \mathsf{75}, \mathsf{200}$ for the gauge group $\mathsf{SU(5)}$ in the expansion of the $(\mathsf{24} \times \mathsf{24})_{\rm sym}$ to the $\mathsf{U(1)_Y}$, $\mathsf{SU(2)_L}$, $\mathsf{SU(3)_C}$ gaugino masses from the VEVS of their respective F-terms.

The numerical analysis of the Yukawa couplings at the electroweak scale which generate the observe quark lepton masses, is done within the framework of supergravity grand unified models using non-universality of gaugino masses.

Non-universal gaugino masses have been discussed extensively in supergravity unified models [42–51]. Since they enter prominently in the analysis below, we give a brief discussion of them before proceeding. They arise in supergravity grand unified theories via dimension 5 operators in the Lagrangian

$$\mathcal{L}_\lambda = \int d^2\theta W_\alpha W_\beta \frac{\Phi_{\alpha\beta}}{M_{\rm Pl}} + h.c. \tag{6.3}$$

A VEV of the F-term of the chiral field Φ leads to mass generation for the gauginos so that

$$\mathcal{L}_\lambda \sim \frac{< F >_{\alpha\beta}}{M_{\rm Pl}} \lambda^\alpha \lambda^\beta + h.c. \tag{6.4}$$

where $M_{\rm Pl}$ is the Planck mass. Thus if F is of size the intermediate scale, the gaugino masses of size relevant for electroweak physics are generated. For $\mathsf{SU(5)}$ where λ^α belong to the $\mathsf{24}$-plet representation of $\mathsf{SU(5)}$. The symmetric product of $\mathsf{24} \times \mathsf{24}$ can be expanded in irreducible representations of $\mathsf{SU(5)}$ so that $(\mathsf{24} \times \mathsf{24})_{\rm sym} = \mathsf{1} + \mathsf{24} + \mathsf{75} + \mathsf{210}$. We note that $< F >_{\alpha\beta}$ is constrained so that it preserves the Standard Model gauge group and so must be a singlet of the SM gauge group. This allows a computation of the contribution of each of the terms above to the gaugino masses and in this case the gaugino masses will in general be a linear combination of contribution from each. One may thus write the gaugino masses m_i where $i = 1, 2, 3$ stand for the gauge group $\mathsf{U(1)_Y}$, $\mathsf{SU(2)_L}$ and $\mathsf{SU(3)_C}$ so that

$$m_i = m_{\frac{1}{2}} \sum_{r=\mathsf{1},\mathsf{24},\mathsf{75},\mathsf{200}} c_r n_i^r \tag{6.5}$$

The n_i^r in this case are listed in table 5. For the $\mathsf{SO(10)}$ case λ^α belongs to the $\mathsf{45}$-plet representation of $\mathsf{SO(10)}$ and the symmetric product of $\mathsf{45} \times \mathsf{45}$ expanded in irreducible representations of $\mathsf{SO(10)}$ is given by

$$(\mathsf{45} \times \mathsf{45})_{\rm sym} = \mathsf{1} + \mathsf{54} + \mathsf{210} + \mathsf{770} \tag{6.6}$$

JHEP07(2025)222

Next expanding $\mathbf{54}, \mathbf{210}, \mathbf{770}$ in irreducible representations of $\mathsf{SU}(5)$, one finds that $\mathbf{54}$-plet contains $\mathbf{24}$-plet of $\mathsf{SU}(5)$, $\mathbf{210}$ contains ($\mathbf{1}$, $\mathbf{24}$, $\mathbf{75}$)-plets of $\mathsf{SU}(5)$ and $\mathbf{770}$-plet contains ($\mathbf{1}$, $\mathbf{24}$, $\mathbf{75}$, $\mathbf{200}$)-plets of $\mathsf{SU}(5)$. Thus we see that for $\mathsf{SO}(10)$ case we may still write the non-univeral gaugino masses exactly as given by eq. (6.5) except that c_r receive contributions now from all the irreducible representations of $\mathsf{SO}(10)$ listed in eq. (6.6). Universal gaugino masses are obtained if we retain only the singlet term in eq. (6.6). However, inclusion of higher dimensional representations in the breaking gives rise to non-universalities since c_r are free parameters which can only be determined in a larger theory. As an illustration let us consider only the F-breaking via the representations $\mathbf{1}$, $\mathbf{24}$, $\mathbf{75}$ in eq. (6.5). In this case the choice $c_1 : c_{24} : c_{75} = 2.75 : 1 : 0.25$ leads to gaugino masses in the ratio $m_1 : m_2 : m_3 = 1 : 1 : 10$, so a gaugino mass hierarchy of $\mathcal{O}(10)$ can be simply generated via use of non-singlet F-term breaking in $\mathsf{SO}(10)$. As noted above this mechanism has been extensively used in generating non-universal gaugino masses in applications in supergravity grand unified models [42–51]. This is the underlying mechanism for generation of nonuniversal masses for the gauginos that is utilized in our phenomenological analysis below.

Returning to $b - t - \tau$ unification, we need to run renormalization group equations and calculate the mass spectrum with **SPheno-4.0.4** [52, 53]. After the RG evolution, the quantities of interest at the electroweak scale are $h_t, h_b, h_\tau, m_{h^0}, \Delta a_\mu, \Omega h^2$, where m_{h^0} is Higgs mass, Δa_μ is muon anomaly defined so that $\Delta a_\mu = \Delta(g_\mu - 2)/2$ and Ωh^2 is the dark matter relic density calculated by **MicrOmega-6.1.15** [54]. For the Higgs mass and the relic density we take the experiment constraints so that

$$m_{h^0} = 125 \pm 2 \text{ GeV} \tag{6.7}$$

$$\Omega h^2 < 0.12. \tag{6.8}$$

Regarding Δa_μ, the recent Fermilab experimental measurement [55] finds a deviation from the standard model result of $\Delta a_\mu = (24.9 \pm 4.8) \times 10^{-10}$ [55, 56]. An important contribution to the muon moment comes from the hadronic vacuum polarization (HVP). The Fermilab analysis uses e^+e^- cross section to hadrons in its evaluation of HVP. The analyses of HVP as given by lattice gauge calculations [57–59] indicate a somewhat smaller deviation from the standard model prediction. However, the situation on the lattice gauge calculations is not fully settled. For that reason we will use the Fermilab Δa_μ constraint but with a two σ corridor. As pointed out early on [60] the light sleptons and light weakinos can contribute to the muon anomalous moment and produce a sizable electroweak correction and thus there exists a connection between the Fermilab experiment and the search for sparticles at the LHC [61–63].

The analysis of $b - t - \tau$ unification is done within the renormalization group analysis of $\tilde{g}$SUGRA [50] with gluino mass driven radiative breaking of the electroweak symmetry. To identify the allowed $\tilde{g}$SUGRA parameters, we scan the parameter space and find models that (1) satisfy all experimental constraints and (2) yield third generation fermion masses arising from eq. (6.2) consistent with data. A direct Monte Carlo (MC) scan of the parameter space is done in the following range

$$m_0 \in (0, 1000)\,\text{GeV} \qquad A_0 \in (-5000, 5000)\,\text{GeV} \tag{6.9}$$

$$m_1 \in (0, 1500)\,\text{GeV} \qquad m_2 \in (0, 1500)\,\text{GeV} \qquad m_3 \in (0, 10000)\,\text{GeV} \tag{6.10}$$

$$\tan\beta \in (0, 30) \qquad m_t \in (172.2, 173.0)\,\text{GeV} \qquad \bar{m}_b(\bar{m}_b) \in (4.15, 4.22)\,\text{GeV} \tag{6.11}$$

With values of $U_{d_{r9}}$ and $V_{d_{r9}}$ provided in tables 3 and 4 and input parameter f^{10_r} in table 6, we find the result of Yukawa couplings of the top, the bottom and the tau using eq. (6.2). In table 7 we exhibit SUGRA parameter sets used in RG analysis consistent with the experimental data on the top and the bottom quark masses and for the tau lepton mass. In table 8 we display the light sparticle mass spectrum consistent with the relic density constraint and with the experimental $g_\mu - 2$ constraint. Here it is seen that the lightest sparticles are the neutralino, the stau, the tau-sneutrino, the first two generation sneutrinos, and the first two generation sleptons. In figure 1 we exhibit the RG evolution of the sparticles from the GUT scale to the electroweak scale in $\tilde{g}$SUGRA model for the input parameters of the SUGRA model (a) of table 7. Here in the left panel, we find that while the sleptons and squarks all start with the same universal m_0 for the scalar mass, the RG evolution splits their masses widely at the electroweak scale. Thus the slepton masses remain in the few hundred GeV region, while the squark masses are driven to high values in the several TeV region due to their color interactions driven by the high gluino mass. The right panel exhibits the RG evolution of the Yukawa couplings from the GUT scale to the electroweak scale. Quite remarkably one finds that there is splitting of the $b - t$ Yukawas as the GUT scale due to the bi-unitary transformations needed to diagonalize the Higgs boson mass as given by eq. (4.5). This splitting of $b - t$ is significant enough to achieve a $b - t - \tau$ unification for values of $\tan\beta$ between $\sim (5 - 10)$ as illustrated in figure 2. Further, as noted already the $\tilde{g}$SUGRA model evolution splits the sparticle spectrum into two distinct categories: a light sparticle spectrum consisting of sleptons and weakinos and a heavy sparticle spectrum consisting of heavy gluino, heavy squarks and heavy Higgses H^0, A^0 and $H^\pm$. This is illustrated in figure 3. Here we illustrate the light spectrum vs heavy spectrum split for the case of two models (a) and (c) of table 7 where the upper panels are for case (a) and the lower panels are for case (c). The left panel in each case exhibits the scale of splitting of the light and heavy mass spectrum, while the right panel focuses on the mass hierarchy of the light mass spectrum. The right panels are of special interest for the possible discovery of sparticles at the LHC because of their relatively low masses and there is the possibility that one or two of these sparticles may be accessible at the LHC in future runs.

As noted above because of the split sparticle spectrum in the SO(10) model with $\tilde{g}$SUGRA RG running, possibility exists for the discovery of light sparticles at the LHC. To make it more concrete we estimate now the production cross sections for some of the light sparticles at the LHC. Thus the production cross sections for the first and the second generation sleptons in pp collisions, i.e., $\sigma(pp \to \tilde{e}_L\tilde{e}_L)$, $\sigma(pp \to \tilde{e}_R\tilde{e}_R)$, $\sigma(pp \to \tilde{\mu}_L\tilde{\mu}_L)$, $\sigma(pp \to \tilde{\mu}_R\tilde{\mu}_R)$, and $\sigma(pp \to \tilde{\nu}_\mu\tilde{\nu}_\mu)$ (while $\sigma(pp \to \tilde{\nu}_e\tilde{\nu}_e)$ is similar in size as for the μ case) at $\sqrt{s} = 14\,\text{TeV}$ and $\sqrt{s} = 27\,\text{TeV}$, are given in table 9. In table 10 we give the production cross-sections for the staus, and for the stau-neutrinos, i.e., for $\sigma(pp \to \tilde{\tau}_1\tilde{\tau}_1)$,$\sigma(pp \to \tilde{\tau}_2\tilde{\tau}_2)$, $\sigma(pp \to \tilde{\tau}_1\tilde{\tau}_2)$,

Model	$f^{(10_r)}$	h_t^0	h_b^0	h_τ^0
(a)	$(0.707-0.283i,-0.954-0.019i)$	0.484	0.051	0.051
(b)	(0.567,0.046)	0.498	0.033	0.033
(c)	(0.475,0.103)	0.486	0.046	0.046
(d)	$(1.09+0.918i,-0.013+0.936i)$	0.503	0.050	0.050
(e)	$(0.570+0.379i,1.12-0.362i)$	0.486	0.062	0.062
(f)	$(0.218,0.213)$	0.487	0.056	0.056
(g)	$(0.595+0.196i,0.781-0.063i)$	0.485	0.044	0.044
(h)	$(0.459,0.056)$	0.489	0.043	0.043
(i)	$(1.08+0.901i,-0.0062+0.927i)$	0.485	0.043	0.043
(j)	$(0.566+0.378i,1.12-0.360i)$	0.484	0.055	0.055

Table 6. Input parameter f^{10_r} and the corresponding Yukawa couplings of top, bottom and tau given by eq. (6.2).

$\sigma(pp\to\tilde\nu_\tau\tilde\nu_\tau)$, and $\sigma(pp\to\tilde\tau_1\tilde\nu_\tau)$, at $\sqrt{s}=14\,\text{TeV}$ and $\sqrt{s}=27\,\text{TeV}$. Table 11 gives a display of the branching ratios of $\tilde\tau_1$, $\tilde\tau_2$ and $\tilde\nu_\tau$ exhibiting the fact that the stau-production and their subsequent decays produce a rich array of signatures. (For recent works on stau phenomenology see [56, 64, 65] and for related earlier works see [66–68]. One of the most interesting signatures in supersymmetry is the trileptonic signature arising from the final state $\chi_2^0\chi_1^\pm$ [69–75]. Here the decay of the $\chi_2^0\to\ell^+\ell^-\chi_1^0$ and $\chi_1^\pm\to\ell^\pm+\nu(\bar\nu)+\chi_1^0$ will lead to the trileptonic signature $\chi_2^0\chi_1^\pm\to\ell_1^+\ell_2^+\ell_2^-+\text{missing energy}$. Typically this signal has a low background from the standard model processes. In the analysis of the low scale mass spectrum one finds that the masses of $\chi_2^0\chi_1^\pm$ lie in the few hundred GeV region. It is of interest to estimate the size of the production cross section $\sigma(pp\to\chi_2^0\chi_1^\pm)$. The estimate of the sizes for the chosen benchmarks are given in table 12. From the table one finds the $\sigma(pp\to\chi_2^0\chi_1^\pm)$ cross section can be as large as $\sim 20\,\text{fb}$ for some entries which implies that HL-LHC with an integrated luminosity of $3000\,\text{fb}^{-1}$ will produce $\sim 6\times10^4$ events. There is then a good chance that some of these event may survive the stringent cuts at the LHC and be detectable. A more detailed analysis of signal events is outside the scope of this work but is of interest for further analysis.

7 Conclusion

In this work we have investigated the grand unified group $\mathsf{SO}(10)$ with the Higgs representations $\mathsf{560}+\overline{\mathsf{560}}$ in the heavy Higgs sector and $2\times\mathsf{10}+\mathsf{320}$ in the light sector which has the property that it simultaneously reduces the rank of $\mathsf{SO}(10)$ from 5 to 4 as well as reduces the rest of the symmetry to the Standard Model gauge group. Further, $\mathsf{560}+\overline{\mathsf{560}}$ contain five color Higgs triplets/anti-triplets and only four Higgs doublet pairs while $2\times\mathsf{10}+\mathsf{320}$ light Higgs contain 5 Higgs color triplets/anti-triplets and 5 Higgs doublet pairs. Thus the mixing of the heavy and the light Higgs sectors leads to all the color triplets/anti-triplets becoming heavy and all of the Higgs doublets becoming heavy except one Higgs doublet pair needed for electroweak symmetry breaking. We now list the new elements of the work accomplished in this paper.

Model	m_0	A_0	m_1	m_2	m_3	$\tan\beta$	m_t	$\bar{m_b}(\bar{m_b})$	m_τ
(a)	533.0	-2502	814	815	7945	9.7	172.4	4.15	1.77682
(b)	363.7	-473	740	766	6722	5.9	172.9	4.20	1.77682
(c)	464.2	-3681	561	676	6616	8.6	172.6	4.15	1.77682
(d)	398.8	-1759	859	1111	8566	9.2	172.7	4.21	1.77682
(e)	486.4	1233	882	876	7514	11.6	172.6	4.20	1.77682
(f)	568.0	831	896	733	7987	10.4	172.8	4.17	1.77682
(g)	373.4	-3413	674	1013	7732	8.1	172.2	4.19	1.77682
(h)	497.9	2901	894	759	7589	7.9	172.6	4.22	1.77682
(i)	433.7	930	762	833	7653	8.1	172.2	4.16	1.77682
(j)	624.8	4216	764	742	8836	10.2	172.4	4.19	1.77682

Table 7. The SUGRA parameters sets used for renormalization group analysis of masses of the top, the bottom and the τ lepton, using their Yukawa couplings at the GUT scale as input as given in table (6).

Model	h_0	$\tilde{\chi}_1^0$	$\tilde{\tau}_1$	$\tilde{\nu_\tau}$	$\tilde{\nu_\mu}$	$\tilde{\mu}$	$\tilde{\tau}_2$	$\tilde{\chi}_1^\pm$	$\tilde{\chi}_2^0$	$\tilde{\chi}_2^\pm$	Ωh^2	$\Delta a_\mu(\times 10^{-9})$
(a)	124.4	313	331	410	449	456	628	575	575	7993	0.0643	2.15
(b)	123.3	283	284	325	334	343	490	543	543	6772	0.0183	3.18
(c)	123.7	206	229	326	379	387	519	473	473	7072	0.1220	2.81
(d)	124.2	330	339	466	495	500	590	824	824	8418	0.0146	2.65
(e)	124.3	344	366	460	483	489	627	630	630	7116	0.0937	1.92
(f)	124.8	347	361	419	440	447	673	498	497	7602	0.0538	2.05
(g)	123.1	251	273	416	455	460	526	752	752	7965	0.0771	3.07
(h)	124.0	346	347	386	409	417	596	521	521	7099	0.0240	1.93
(i)	123.7	288	296	358	373	381	551	588	588	7303	0.0281	3.72
(j)	124.1	280	303	370	436	443	642	487	487	8016	0.1210	2.86

Table 8. Low scale SUSY mass spectrum showing the Higgs boson h^0 mass consistent with the constraint of eq. (6.7), and the masses of $\tilde{\chi}_1^0$, $\tilde{\tau}_1$, $\tilde{\nu_\tau}$, $\tilde{\nu_\mu}$, $\tilde{\mu}$, $\tilde{\tau}_2$, $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_2^\pm$ for the set of benchmarks (a)-(j) of table 7. The corresponding relic density Ωh^2 consistent with the constraint of eq. (6.8) and the muon anomaly $\Delta a_\mu(\times 10^{-9})$ consistent with the muon anomaly constraint is also exhibited.

- While the model was proposed in [31] further development of the model required explicit computations of the couplings which involves construction of the 320-multiplet and its couplings to $560 + \overline{560}$ which were not available in previous works. This has been accomplished in the current paper. We expand further on the specifics of the new work below.

- Construction of the irreducible tensor-spinor $|\Theta_{\mu\nu}^{(560)}\rangle$ from the 720 $(= 16 \times 45)$ dimensional reducible tensor-spinor $|\Theta_{\mu\nu}^{720}\rangle$. It is carried out by removing 160 $(= 16 \times 10)$ components from $|\Theta_{\mu\nu}^{720}\rangle$ as shown in appendix A.

- Construction of two 320 multiplets, one symmetric $\left(\Lambda_{(\mu\nu)\lambda}^{(320_s)}\right)$ and the other antisymmetric $\left(\Lambda_{[\mu\nu]\lambda}^{(320_a)}\right)$ in the two indices, starting with the 1000 dimensional reducible tensor

Model	$\sigma(pp \to \tilde{e}_L \tilde{e}_L)$		$\sigma(pp \to \tilde{\mu}_L \tilde{\mu}_L)$		$\sigma(pp \to \tilde{e}_R \tilde{e}_R)$		$\sigma(pp \to \tilde{\mu}_R \tilde{\mu}_R)$		$\sigma(pp \to \tilde{\nu}_e \tilde{\nu}_e)$		$\sigma(pp \to \tilde{\nu}_\mu \tilde{\nu}_\mu)$	
	14 TeV	27 TeV	14 TeV	27 TeV	14 TeV	27 TeV	14 TeV	27 TeV	14 TeV	27 TeV	14 TeV	27 TeV
(a)	0.936	3.619	0.940	3.632	0.077	0.382	0.078	0.382	0.932	3.621	0.933	3.625
(b)	3.208	10.577	3.216	10.603	0.317	1.236	0.316	1.235	3.358	1.110	3.359	1.111
(c)	1.920	6.738	1.931	6.771	0.179	0.762	0.179	0.763	1.964	6.912	1.968	6.931
(d)	0.572	2.379	0.570	2.372	0.343	1.323	0.345	1.330	0.560	2.344	5.598	2.344
(e)	0.677	2.745	0.680	2.757	0.106	0.491	0.105	0.491	0.668	2.722	0.668	2.724
(f)	1.022	3.903	1.025	3.913	0.060	0.309	0.060	0.309	1.020	3.916	1.021	3.918
(g)	0.883	3.442	0.899	3.498	0.278	1.106	0.274	1.093	0.877	3.436	0.878	3.440
(h)	1.396	5.105	1.399	5.117	0.101	0.472	0.101	0.472	1.408	5.176	1.410	5.180
(i)	2.049	7.131	2.056	7.154	0.185	0.782	0.184	0.782	2.102	7.345	2.103	7.349
(j)	1.066	4.046	1.070	4.061	0.051	0.271	0.051	0.271	1.065	4.065	1.068	4.073

Table 9. The NLO+NNLL pair production cross-sections, in fb, of first and second generation sleptons at $\sqrt{s} = 14$ TeV and 27 TeV for all benchmark models.

Model	$\sigma(pp \to \tilde{\tau}_1 \tilde{\tau}_1)$		$\sigma(pp \to \tilde{\tau}_2 \tilde{\tau}_2)$		$\sigma(pp \to \tilde{\tau}_1 \tilde{\tau}_2)$		$\sigma(pp \to \tilde{\nu}_\tau \tilde{\nu}_\tau)$		$\sigma(pp \to \tilde{\tau}_1 \tilde{\nu}_\tau)$	
	14 TeV	27 TeV	14 TeV	27 TeV	14 TeV	27 TeV	14 TeV	27 TeV	14 TeV	27 TeV
(a)	2.584	8.335	0.076	0.369	1.398	5.144	0.112	0.459	0.634	2.427
(b)	5.065	15.29	0.246	0.981	3.745	12.238	0.266	0.950	1.281	4.429
(c)	10.171	28.29	0.195	0.810	3.713	12.144	0.376	1.325	2.684	8.903
(d)	0.893	3.011	0.242	1.052	0.637	2.616	0.186	0.733	0.951	3.984
(e)	1.460	4.935	0.080	0.390	0.841	3.316	0.122	0.509	0.551	2.253
(f)	2.008	6.769	0.053	0.275	1.267	4.722	0.060	0.260	0.354	1.397
(g)	3.621	10.649	0.232	0.975	1.311	4.865	0.366	1.333	1.970	7.257
(h)	2.416	7.993	0.098	0.454	1.821	6.477	0.088	0.354	0.430	1.637
(i)	4.142	12.715	0.144	0.625	2.509	8.585	0.191	0.721	1.049	3.755
(j)	4.134	12.835	0.068	0.335	2.184	7.596	0.090	0.364	0.690	2.505

Table 10. The stau and stau-neutrino pair production cross-sections at NLO+NNLL order in fb, at $\sqrt{s} = 14$ TeV and 27 TeV for all benchmark models. The largest cross sections are for the production of $\tilde{\tau}_1 \tilde{\tau}_1$ followed by the production cross section for $\tilde{\tau}_1 \tilde{\tau}_2$.

$T_{\mu\nu\lambda}$. Complete details are carried out in appendix D and results exhibited in eqs. (2.6) and (2.7).

- Couplings involving tensor spinors with tensor Higgs representations are not available in the literature and we have developed new techniques to accomplish it. Thus one of the central accomplishments of this paper is that we have carried out an explicit computation of the couplings of the heavy sector with the light sector, i.e., $(\mathsf{560} \cdot \mathsf{560}) \cdot \mathsf{10}_i$ $(i = 1, 2)$ and $(\mathsf{560} \cdot \mathsf{560}) \cdot \mathsf{320}$ using one of the tensor representations of the $\mathsf{320}$, i.e., the one symmetric in two indices as shown in appendices E, F and G.

- Further, we have computed the mass matrix of the Higgs doublets arising from the $\mathsf{560} + \overline{\mathsf{560}}$, $\mathsf{320}$, and from $\mathsf{10}_i$ $(i = 1, 2)$ after spontaneous breaking of the GUT symmetry in terms of the parameters of the cubic superpotential as shown in appendix G and displayed in eq. (4.4).

JHEP07(2025)222

Model	$\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$	$\tilde{\tau}_2 \to \tau\tilde{\chi}_1^0$	$\tilde{\tau}_2 \to \tilde{\nu}_\tau W^-$	$\tilde{\tau}_2 \to \tilde{\tau}_1 Z[h^0]$	$\tilde{\nu}_\tau \to \tau\tilde{\chi}_1^0$	$\tilde{\nu}_\tau \to \tilde{\tau}_1 W^+$	$\tilde{\nu}_\tau \to \tilde{\tau}_1 q\bar{q}$
(a)	100%	12%	41%	35%[12%]	91%	<1%	9%
(b)	100%	23%	37%	28%[12%]	99%	<1%	1%
(c)	100%	99%	<1%	<1%[<1%]	31%	69%	<1%
(d)	100%	15%	22%	62%[<1%]	9%	91%	<1%
(e)	100%	14%	36%	44%[6%]	32%	68%	<1%
(f)	100%	12%	42%	29% [15%]	98%	<1%	2%
(g)	100%	19%	14%	67%[<1%]	9%	91%	<1%
(h)	100%	98%	<1%	<1%[<1%]	99%	<1%	<1%
(i)	100%	16%	39%	32%[12%]	98%	<1%	2%
(j)	100%	98%	<1%	<1% [<1%]	97%	<1%	3%

Table 11. While the decay branching ratio of $\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$ is 100%, the decay branching ratios of $\tilde{\tau}_2$ and $\tilde{\nu}_\tau$ are diverse and show a variety of model dependent signatures providing differentiation among cases (a)-(j) for this class of $\mathsf{SO}(10)$ models.

Model	$\sigma(pp \to \tilde{\chi}_2^0\tilde{\chi}_1^\pm)$	
	14TeV	27TeV
(a)	8.773	40.105
(b)	11.446	50.159
(c)	21.770	87.013
(d)	0.731	5.284
(e)	5.564	27.362
(f)	17.400	71.840
(g)	2.216	12.828
(h)	13.977	59.533
(i)	7.838	36.461
(j)	19.198	78.215

Table 12. The production cross-section for $\tilde{\chi}_2^0\tilde{\chi}_1^\pm$ in fb at $\sqrt{s} = 14$ TeV and 27 TeV for all models. As discussed in text the process $pp \to \tilde{\chi}_2^0\tilde{\chi}_1^\pm$ is of significant interest and it leads to the well-known tri-leptonic signal for supersymmetry.

- After diagonalization of both the kinetic energy and the mass matrix we have identified the pair of light Higgs doublets that enter in the electroweak symmetry breaking. Thus we have deduced the MSSM with a pair of light Higgs doublets in a natural way from a diagonalization involving nine Higgs doublets where all but the electroweak doublets become superheavy as shown in eq. (4.3) and eq. (4.5).

- As another major result of the paper we show that the diagonalization of the Higgs mass matrix leads to splitting between the couplings of the up Higgs to the top quark vs the coupling of the down Higgs to the bottom quark due to the light Higgs doublets arising as linear combinations of the Higgses in $\mathsf{320}$ and $\mathsf{10}_i$ $(i = 1, 2)$. This allows the Yukawa coupling of the top quark becoming larger than the Yukawa coupling of the down quark at the GUT scale which leads to $b - t - \tau$ unification with low $\tan\beta$ with $\tan\beta$ as low as 5–10 as shown in table 6 and table 7.

JHEP07(2025)222

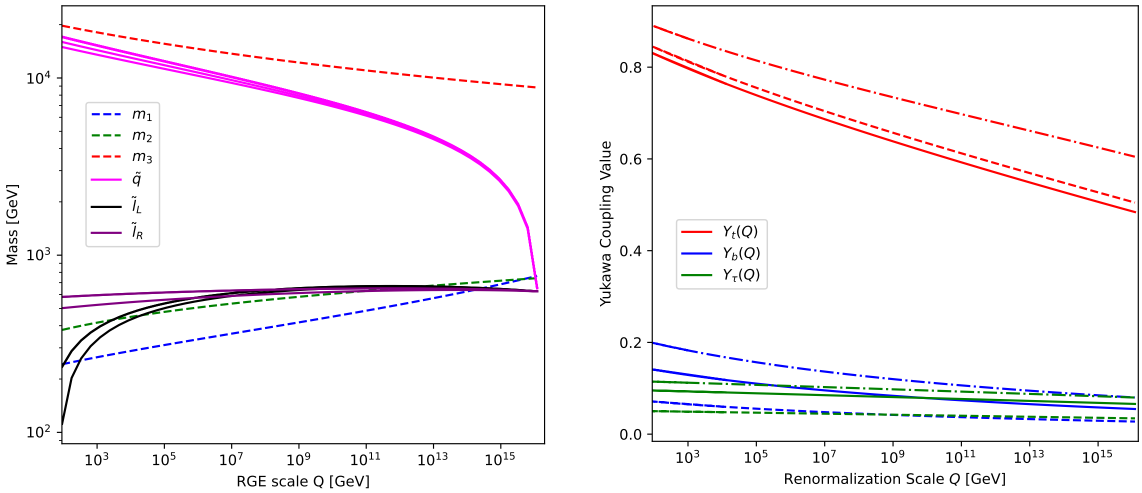

Figure 1. Left panel: renormalization group evolution of sleptons, squarks and of the gaugino masses in $\tilde{g}$SUGRA for model (a). Right panel: renormalization group evolution of Yukawa couplings of b and t quarks and of the τ lepton. The dash line, solid line and dash dot line correspond to $\tan\beta = 5$, 9.8 and 14.

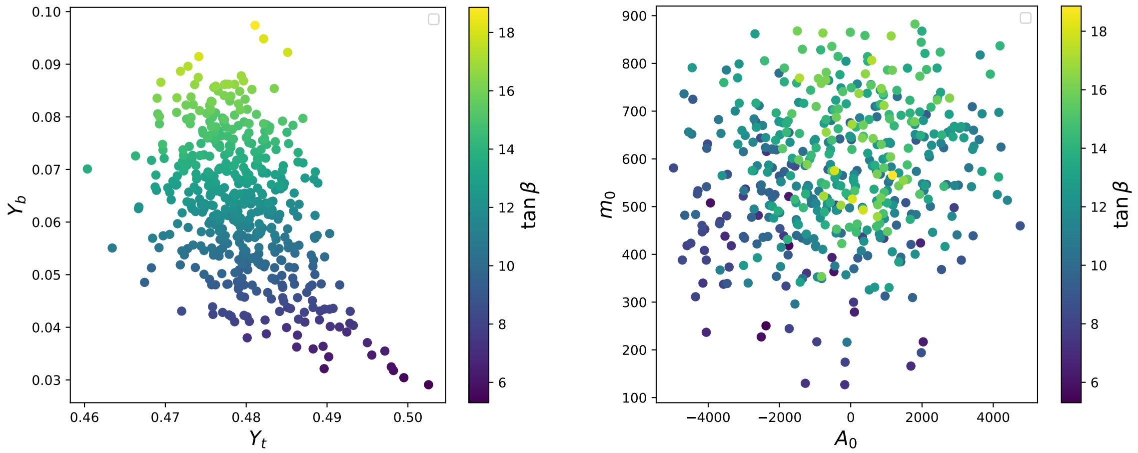

Figure 2. Left panel: an MC scan of the top and the bottom Yukawa couplings at the GUT scale with color map giving $\tan\beta$. Right panel: an MC scan of m_0 vs. A_0 with color map giving $\tan\beta$. All the scatter points satisfy the Higgs boson mass, the relic density and the muon anomaly constraint.

- It is shown that the SO(10) model with $\tilde{g}$SUGRA RG evolution leads to split sparticle mass spectrum, with a set of light sparticles involving the sleptons and the weakinos with masses lying in the few hundred GeV region consistent with the Higgs boson mass, the relic density and $g_\mu - 2$ data constraints. The remaining particle spectrum consisting of the gluino, the squarks and the Higgs $H^0, A^0, H^\pm$ is heavy with masses lying in the TeV region. This is exhibited in table 8 and in figure 3.

- The production cross sections for the light sparticles are estimated in table 9, table 10, and in table 12, and found to be of order few fb which appears encouraging for the

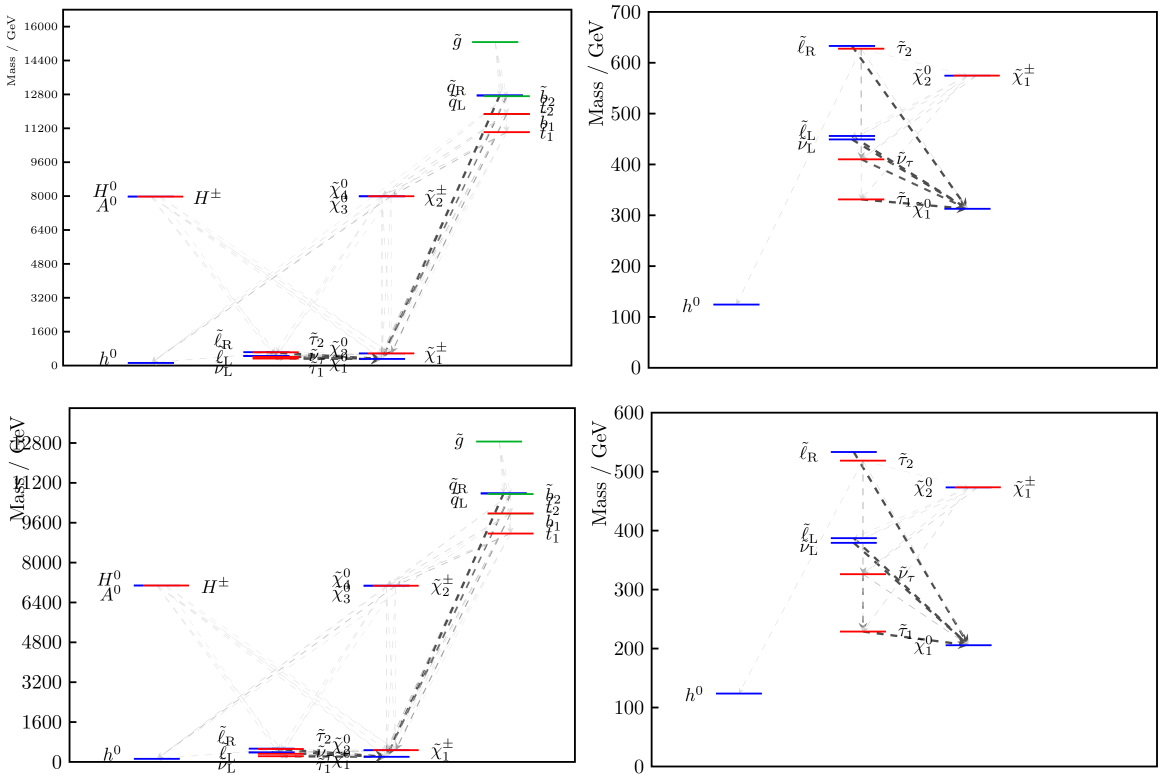

Figure 3. Particle mass spectrum given by **PySLHA** [76] for model (a) (upper panels) and (c) (lower panels). The right panels are zoomed-in to the left panels for low-lying masses.

discovery of supersymmetry at the LHC. Specifically, the $\tilde{\chi}_2^0\tilde{\chi}_1^\pm$ production cross section can lie in the range 40–80 fb as seen in table 12 and is of considerable interest as it leads to the trileptonic signal which is one of the signature events of supersymmetry because it typically has low background from the Standard Model processes. However, a more dedicated analysis is needed to make definitive predictions after the appropriate detector cuts and backgrounds are included in the analysis.

There are significant possibilities for further work in this class of SO(10) models. Thus, as mentioned above, a more dedicated analysis of sparticle production signals taking into account detector cuts and backgrounds, would allow one to make reliable estimates of what one may expect at HL-LHC and HE-LHC. Since all the parameters of the model are now constrained, prediction of proton decay branching ratios into $p \to e^+\pi^0, p \to \bar{\nu}K^+$ and into other decay modes can be made. Another possible extension of the model is a double copy, i.e., MSSM$_2$, which gives four pair of light Higgs. In this case the possibility that one pair of light Higgs doublets couple with the third generation of fermions and the other to the first two generation of fermions has the potential of explaining the relative mass hierarchy between the third generation and the first generation of fermions. Of course such an extension requires a check on gauge coupling unification and consistency with the set of experimental constraints discussed in the analysis of this work.

Acknowledgments

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A $\mathsf{560}+\overline{\mathsf{560}}$ tensor-spinor in its $\mathsf{SU(5)}$ oscillator modes

We begin by displaying the tensorial structure of each $\mathsf{SU(5)}$ multiplets within the $\mathsf{560}-$plet.

$$\begin{aligned}\mathsf{560}\left[\Theta^{\check{s}(560)}_{\mu\nu}\right] = \mathsf{1}\left[\mathsf{H}^{(560)}\right] + \overline{\mathsf{5}}\left[\mathsf{H}^{(560)}_{i}\right] + \overline{\mathsf{10}}\left[\mathsf{H}^{(560)}_{ij}\right] + \mathsf{10}\left[\mathsf{H}^{(560)ij}\right] + \mathsf{10}\left[\widehat{\mathsf{H}}^{(560)ij}\right]\\ +\mathsf{24}\left[\mathsf{H}^{(560)i}_{j}\right] + \mathsf{40}\left[\mathsf{H}^{(560)ijk}_{l}\right] + \mathsf{45}\left[\mathsf{H}^{(560)ij}_{k}\right] + \overline{\mathsf{45}}\left[\mathsf{H}^{(560)k}_{ij}\right]\\ +\overline{\mathsf{50}}\left[\mathsf{H}^{(560)lm}_{ijk}\right] + \overline{\mathsf{70}}\left[\mathsf{H}^{(560)k}_{(S)ij}\right] + \mathsf{75}\left[\mathsf{H}^{(560)ij}_{kl}\right] + \mathsf{175}\left[\mathsf{H}^{(560)m}_{[ijk]l}\right],\end{aligned} \tag{A.1}$$

where the Latin indices $i,j,k,\dots$ taking on the values $1,2,\dots,5$. From here onward, we suppress the spinor index $\check{s}$ for brevity.

A.1 The reducible 720 dimensional tensor-spinor multiplet

In eqs. (2.5) it was shown that the irreducible tensor-spinor $|\Theta^{(560)}_{\mu\nu}\rangle$ can be obtained from the $\mathsf{720}$ $(=\mathsf{16}\times\mathsf{45})$ dimensional reducible tensor-spinor $|\Theta^{720}_{\mu\nu}\rangle$ by removing $\mathsf{160}$ $(=\mathsf{16}\times\mathsf{10})$ components. From here onward, we suppress the spinor index for brevity. For the purpose of applications we need to exhibit the decomposition of reducible 720 and irreducible $\mathsf{560}$ mulitplets in terms of the $\mathsf{SU(5)}$ modes. To this end we consider a field theoretic description of the $\mathsf{720}-$plet and define the Clifford elements Γ_μ in terms of creation and annihilation operators b_i and $b_i^\dagger$ [36, 37] as $\Gamma_{2i}=(b_i+b_i^\dagger), \Gamma_{2i-1}=-i(b_i-b_i^\dagger)$, where $\{b_i,b_j^\dagger\}=\delta_{ij}$, $\{b_i,b_j\}=0$ and $\{b_i^\dagger,b_j^\dagger\}=0$ and that the $\mathsf{SU(5)}$ singlet satisfies $b_i\,|0\rangle=0$. In terms of $b_i,b_i^\dagger$ the $\mathsf{16}$ and $\overline{\mathsf{16}}$ multiplets of $\mathsf{SO(10)}$ have the following $\mathsf{SU(5)}$ oscillator expansions: $|\Theta^{(16)}\rangle = |0\rangle\,M+\frac{1}{2}b_i^\dagger b_j^\dagger\,|0\rangle\,M^{ij}+\frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger\,|0\rangle\,M_i$ and $|\overline{\Theta}^{(16)}\rangle = b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_4^\dagger\,|0\rangle\,N+\frac{1}{12}\epsilon^{ijklm}b_k^\dagger b_l^\dagger b_m^\dagger\,|0\rangle\,N_{ij}+b_i^\dagger\,|0\rangle\,N^i$. Thus we have

$$|\Theta^{(720)}_{\mu\nu}\rangle = |0\rangle\,X_{\mu\nu}+\frac{1}{2}b_i^\dagger b_j^\dagger\,|0\rangle\,X^{ij}_{\mu\nu}+\frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger\,|0\rangle\,X_{i\mu\nu}. \tag{A.2}$$

We now display the $\mathsf{SU(5)}$ decomposition of $|\Theta^{(720)}_{\mu\nu}\rangle$:

$$\begin{aligned}\Theta_{\mu\nu}(720) &= X_{\mu\nu}(45)\oplus X_{i\mu\nu}(\overline{5}\times 45)\oplus X^{ij}_{\mu\nu}(10\times 45),\\ X_{\mu\nu}(45) &= X_{c_ic_j}(10)\oplus X_{\bar{c}_i\bar{c}_j}(\overline{10})\oplus X_{c_i\bar{c}_j}(25),\\ X_{i\mu\nu}(\overline{5}\times 45) &= X_{ic_jc_k}(\overline{5}\times 10)\oplus X_{i\bar{c}_j\bar{c}_k}(\overline{5}\times\overline{10})\oplus X_{ic_k\bar{c}_j}(\overline{5}\times 25),\\ X^{ij}_{\mu\nu}(10\times 45) &= X^{ij}_{c_kc_l}(10\times 10)\oplus X^{ij}_{\bar{c}_k\bar{c}_l}(10\times\overline{10})\oplus X^{ij}_{c_k\bar{c}_l}(10\times 25),\end{aligned} \tag{A.3}$$

where the quantities in the parenthesis represent the dimensionality of the tensor. These $\mathsf{SU(5)}$ reducible tensors are further decomposed into their irreducible parts as follows

$$\begin{aligned}\mathsf{10}_1:&\quad X_{c_ic_j} = {}^{(1)}U^{ij},\\ \overline{\mathsf{10}}:&\quad X_{\bar{c}_i\bar{c}_j} = U_{ij},\\ \mathsf{25}=\mathsf{24}_1+\mathsf{1}_1:&\quad X_{c_i\bar{c}_j} = {}^{(1)}U^i_j+\frac{1}{5}\delta^i_j\,{}^{(1)}U,\end{aligned} \tag{A.4}$$

$$\overline{5}\times 10 = 45+5: \quad X_{ic_jc_k} = U_i^{jk} + \frac{1}{4}\left(\delta_i^k U^j - \delta_i^j U^k\right),$$
$$\overline{5}\times\overline{10} = 40_1+10_2: \quad X_{i\bar{c}_j\bar{c}_k} = \epsilon_{jklmn}{}^{(1)}U_i^{lmn} + \epsilon_{ijklm}{}^{(2)}U^{lm},$$
$$\overline{5}\times 25 = \overline{45_1}+\overline{5}_1+\overline{70}+\overline{5_2}: \quad X_{ic_k\bar{c}_j} = \frac{1}{2}{}^{(1)}U_{ij}^k + \frac{1}{8}\left(\delta_j^{k\,(1)}U_i - \delta_i^{k\,(1)}U_j\right) + \frac{1}{2}U_{(S)ij}^k + \frac{1}{12}\left(\delta_j^{k\,(2)}U_i - \delta_i^{k\,(2)}U_j\right), \tag{A.5}$$

$$\begin{aligned}
10\times 10 = \overline{50}+\overline{45_2}+\overline{5_3}: \quad X_{c_kc_l}^{ij} &= \epsilon^{ijmno}U_{mno}^{kl} + \frac{3}{2}\left(\epsilon^{ijkmn\,(2)}U_{mn}^l - \epsilon^{ijlmn\,(2)}U_{mn}^k\right) + \frac{1}{2}\epsilon^{ijklm\,(3)}U_m,\\
10\times\overline{10} = 75+24_2+1_2: \quad X_{\bar{c}_k\bar{c}_l}^{ij} &= U_{kl}^{ij} + \frac{1}{3}\left(\delta_l^{i\,(2)}U_k^j - \delta_k^{i\,(2)}U_l^j + \delta_k^{j\,(2)}U_l^i - \delta_l^{j\,(2)}U_k^i\right) + \frac{1}{20}\left(\delta_l^i\delta_k^j - \delta_k^i\delta_l^j\right){}^{(2)}U,\\
10\times 25 = 10_3+15+10_4+40_2+175: \quad X_{c_k\bar{c}_l}^{ij} &= \frac{1}{6}\left(\delta_l^{i\,(3)}U^{jk} - \delta_l^{j\,(3)}U^{ik} + \delta_l^{k\,(3)}U^{ij}\right)\\
&\quad + \frac{1}{8}\left(\delta_l^i U_{(S)}^{jk} - \delta_l^j U_{(S)}^{ik}\right)\\
&\quad + \frac{1}{3}\left(\delta_l^{i\,(4)}U^{jk} - \delta_l^{j\,(4)}U^{ik} + \delta_l^{k\,(4)}U^{ij}\right)\\
&\quad + {}^{(2)}U_l^{ijk} + \epsilon^{ijxyz}U_{[xyz]l}^k.
\end{aligned} \tag{A.6}$$

Next, we investigate the implication of the constraint eq. (2.5a). To that end, contracting Γ_μ with the 720−plet gives

$$\Gamma_\mu\,|\Theta_{\mu\nu}^{(720)}\rangle = |\overline{\Theta}_\nu^{(160)}\rangle\,, \tag{A.7}$$

where

$$|\overline{\Theta}_{c_n}^{(160)}\rangle = b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_4^\dagger|0\rangle U^n + b_i^\dagger|0\rangle\left({}^{(1)}U^{\text{in}} - \frac{1}{2}{}^{(3)}U^{\text{in}} - \frac{1}{2}U_{(S)}^{\text{in}} - {}^{(4)}U^{\text{in}}\right) + \frac{1}{12}\epsilon^{ijklm}b_k^\dagger b_l^\dagger b_m^\dagger|0\rangle\left[72{}^{(2)}U_{ij}^n + 18\left(\delta_i^{n\,(3)}U_j - \delta_j^{n\,(3)}U_i\right) + {}^{(1)}U_{ij}^n + \frac{1}{4}\left(\delta_i^{n\,(1)}U_j - \delta_j^{n\,(1)}U_i\right)\right] \tag{A.8a}$$

$$|\overline{\Theta}_{\bar{c}_n}^{(160)}\rangle = b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_4^\dagger|0\rangle\left(-\frac{1}{2}{}^{(1)}U_n - \frac{1}{2}{}^{(2)}U_n\right) + b_i^\dagger|0\rangle\left({}^{(1)}U_n^i + \frac{1}{5}\delta_n^{i\,(1)}U - {}^{(2)}U_n^i - \frac{1}{5}\delta_n^{i\,(2)}U\right) + \frac{1}{12}\epsilon^{ijklm}b_k^\dagger b_l^\dagger b_m^\dagger|0\rangle\left[\epsilon_{ijnop}\left({}^{(2)}U^{op} + 3{}^{(3)}U^{op} + 6{}^{(4)}U^{op}\right) + \epsilon_{ijopq}\left({}^{(1)}U_n^{opq} + 6{}^{(2)}U_n^{opq}\right)\right] \tag{A.8b}$$

To get the 560 tensor-spinor, $|\Theta_{\mu\nu}^{(560)}\rangle$, we must impose eq. (2.5a). This is equivalent to setting

$|\Theta^{(\overline{160})}_{\nu}\rangle = 0$. Thus eqs. (A.8) gives the following constraints

$$U^{n} = 0, \tag{A.9a}$$

$$^{(1)}U^{\text{in}} - \frac{1}{2}{}^{(3)}U^{\text{in}} - \frac{1}{2}U^{\text{in}}_{(S)} - {}^{(4)}U^{\text{in}} = 0, \tag{A.9b}$$

$$72\,{}^{(2)}U^{n}_{ij} + 18\left(\delta^{n}_{i}{}^{(3)}U_{j} - \delta^{n}_{j}{}^{(3)}U_{i}\right) + {}^{(1)}U^{n}_{ij} + \frac{1}{4}\left(\delta^{n}_{i}{}^{(1)}U_{j} - \delta^{n}_{j}{}^{(1)}U_{i}\right) = 0, \tag{A.9c}$$

$$-\frac{1}{2}{}^{(1)}U_{n} - \frac{1}{2}{}^{(2)}U_{n} = 0, \tag{A.9d}$$

$$^{(1)}U^{i}_{n} + \frac{1}{5}\delta^{i}_{n}{}^{(1)}U - {}^{(2)}U^{i}_{n} - \frac{1}{5}\delta^{i}_{n}{}^{(2)}U = 0, \tag{A.9e}$$

$$\epsilon_{ijnop}\left({}^{(2)}U^{op} + 3\,{}^{(3)}U^{op} + 6\,{}^{(4)}U^{op}\right) + \epsilon_{ijopq}\left({}^{(1)}U^{opq}_{n} + 6\,{}^{(2)}U^{opq}_{n}\right) = 0. \tag{A.9f}$$

The above allows us to write

$$|\Theta^{(\overline{560})}_{\mu\nu}\rangle = |\Theta^{(\overline{720})}_{\mu\nu}\rangle\Big|_{constraint\ of\ eqs.\ (A.9)} \tag{A.10}$$

Next we replace the notation U, U_i, U_{ij} etc by the more tranparent particle notation by defining

$^{(2)}U \equiv \mathsf{H}^{(\overline{560})}$, $\quad {}^{(1)}U_{i} \equiv \mathsf{H}^{(\overline{560})}_{i}$, $\quad U_{ij} \equiv \mathsf{H}^{(\overline{560})}_{ij}$, $\quad {}^{(1)}U^{ij} \equiv \mathsf{H}^{(\overline{560})ij}$, $\quad {}^{(2)}U^{ij} \equiv \widehat{\mathsf{H}}^{(\overline{560})ij}$,
$^{(2)}U^{i}_{j} \equiv \mathsf{H}^{(\overline{560})i}_{j}$, $\quad {}^{(1)}U^{ijk}_{l} \equiv \mathsf{H}^{(\overline{560})ijk}_{l}$,
$U^{ij}_{k} \equiv \mathsf{H}^{(\overline{560})ij}_{k}$, $\quad {}^{(1)}U^{k}_{ij} \equiv \mathsf{H}^{(\overline{560})k}_{ij}$, $\quad U^{lm}_{ijk} \equiv \mathsf{H}^{(\overline{560})lm}_{ijk}$, $\quad U^{k}_{(S)ij} = \mathsf{H}^{(\overline{560})k}_{(S)ij}$, $\quad U^{ij}_{kl} = \mathsf{H}^{(\overline{560})ij}_{kl}$,
$U^{m}_{[ijk]l} = \mathsf{H}^{(\overline{560})m}_{[ijk]l}$.

With the above notation and using eqs. (A.9), we write the expansion of the $\overline{560}$−dimensional tensor-spinor in terms of its SU(5) oscillator modes:

$$\begin{aligned}|\Theta^{(\overline{560})}_{c_x c_y}\rangle =& |0\rangle\,\mathsf{H}^{(\overline{560})xy} + \frac{1}{2}b^{\dagger}_{i}b^{\dagger}_{j}|0\rangle\left[\epsilon^{ijklm}\mathsf{H}^{(\overline{560})xy}_{klm}\right.\\ &\left.+\frac{1}{48}\left(\epsilon^{ijkly}\mathsf{H}^{(\overline{560})x}_{kl} - \epsilon^{ijklx}\mathsf{H}^{(\overline{560})y}_{kl}\right) - \frac{1}{144}\epsilon^{ijkxy}\mathsf{H}^{(\overline{560})}_{k}\right]\\ &+\frac{1}{24}\epsilon^{ijklm}b^{\dagger}_{j}b^{\dagger}_{k}b^{\dagger}_{l}b^{\dagger}_{m}|0\rangle\,\mathsf{H}^{(\overline{560})xy}_{i},\end{aligned} \tag{A.11}$$

$$\begin{aligned}|\Theta^{(\overline{560})}_{\bar{c}_x \bar{c}_y}\rangle =& |0\rangle\,\mathsf{H}^{(\overline{560})}_{xy} + \frac{1}{2}b^{\dagger}_{i}b^{\dagger}_{j}|0\rangle\left[\mathsf{H}^{(\overline{560})ij}_{xy} + \frac{1}{3}\left(\delta^{i}_{y}\mathsf{H}^{(\overline{560})j}_{x} - \delta^{i}_{x}\mathsf{H}^{(\overline{560})j}_{y} + \delta^{j}_{x}\mathsf{H}^{(\overline{560})i}_{y} - \delta^{j}_{y}\mathsf{H}^{(\overline{560})i}_{x}\right)\right.\\ &\left.+\frac{1}{20}\left(\delta^{i}_{y}\delta^{j}_{x} - \delta^{i}_{x}\delta^{j}_{y}\right)\mathsf{H}^{(\overline{560})}\right] + \frac{1}{24}\epsilon^{ijklm}b^{\dagger}_{j}b^{\dagger}_{k}b^{\dagger}_{l}b^{\dagger}_{m}|0\rangle\left[\epsilon_{nopxy}\mathsf{H}^{(\overline{560})nop}_{i}\right.\\ &\left.+\epsilon_{inoxy}\widehat{\mathsf{H}}^{(\overline{560})no}\right],\end{aligned} \tag{A.12}$$

$$\begin{aligned}|\Theta^{(\overline{560})}_{c_x \bar{c}_y}\rangle =& |0\rangle\left[\mathsf{H}^{(\overline{560})x}_{y} + \frac{1}{5}\delta^{x}_{y}\mathsf{H}^{(\overline{560})}\right] + \frac{1}{2}b^{\dagger}_{i}b^{\dagger}_{j}|0\rangle\left[\frac{1}{4}\left(\delta^{i}_{y}\mathsf{H}^{(\overline{560})jx} - \delta^{j}_{y}\mathsf{H}^{(\overline{560})ix}\right)\right.\\ &\left.-\frac{1}{72}\left(4\delta^{x}_{y}\widehat{\mathsf{H}}^{(\overline{560})ij} - \delta^{j}_{y}\widehat{\mathsf{H}}^{(\overline{560})ix} + \delta^{i}_{y}\widehat{\mathsf{H}}^{(\overline{560})jx}\right) - \frac{1}{6}\mathsf{H}^{(\overline{560})ijx}_{y} + \epsilon^{ijklm}\mathsf{H}^{(\overline{560})x}_{[klm]y}\right]\\ &+\frac{1}{24}\epsilon^{ijklm}b^{\dagger}_{j}b^{\dagger}_{k}b^{\dagger}_{l}b^{\dagger}_{m}|0\rangle\left[\frac{1}{24}\left(5\delta^{x}_{y}\mathsf{H}^{(\overline{560})}_{i} - \delta^{x}_{i}\mathsf{H}^{(\overline{560})}_{y}\right) - \frac{1}{2}\left(\mathsf{H}^{(\overline{560})x}_{iy} + \mathsf{H}^{(\overline{560})x}_{(S)iy}\right)\right].\end{aligned} \tag{A.13}$$

Similarly, one can start with $\overline{720}$ $(= \overline{16} \times 45)$ tensor-spinor as

$$|\overline{\Theta}^{(\overline{720})}_{\mu\nu}\rangle = b^{\dagger}_{1}b^{\dagger}_{2}b^{\dagger}_{3}b^{\dagger}_{4}b^{\dagger}_{4}\,|0\rangle\,\overline{X}_{\mu\nu} + \frac{1}{12}\epsilon^{ijklm}b^{\dagger}_{k}b^{\dagger}_{l}b^{\dagger}_{m}\,|0\rangle\,\overline{X}_{\mu\nu ij} + b^{\dagger}_{i}\,|0\rangle\,\overline{X}^{i}_{\mu\nu},$$

and use the constraint $\Gamma_\mu\,|\overline{\Theta}^{(720)}_{\mu\nu}\rangle = |\Theta^{(160)}_{\nu}\rangle = 0$ (equivalent to $\Gamma_\mu\,|\overline{\Theta}^{(560)}_{\mu\nu}\rangle = 0$) to obtain the $\overline{560}$−plet. Here we find

$$
\begin{aligned}
|\overline{\Theta}^{(\mathsf{560})}_{c_x c_y}\rangle &= b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger\,|0\rangle\,\mathsf{H}^{(\mathsf{560})xy} + \frac{1}{12}\epsilon^{ijklm} b_k^\dagger b_l^\dagger b_m^\dagger\,|0\rangle \left[\mathsf{H}^{(\mathsf{560})xy}_{ij} + \frac{1}{3}\left(\delta_i^y \mathsf{H}^{(\mathsf{560})x}_j - \delta_i^x \mathsf{H}^{(\mathsf{560})y}_j\right.\right.\\
&\quad \left.\left. +\delta_j^x \mathsf{H}^{(\mathsf{560})y}_i - \delta_j^y \mathsf{H}^{(\mathsf{560})x}_i\right) + \frac{1}{20}\left(\delta_i^y\delta_j^x - \delta_i^x\delta_j^y\right)\mathsf{H}^{(\mathsf{560})}\right]\\
&\quad +b_i^\dagger\,|0\rangle\left[\epsilon^{nopxy}\mathsf{H}^{(\mathsf{560})i}_{nop} + \epsilon^{inoxy}\widehat{\mathsf{H}}^{(\mathsf{560})}_{no}\right], \qquad \text{(A.14)}\\
|\overline{\Theta}^{(\mathsf{560})}_{\bar c_x \bar c_y}\rangle &= b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger\,|0\rangle\,\mathsf{H}^{(\mathsf{560})}_{xy} + \frac{1}{12}\epsilon^{ijklm} b_k^\dagger b_l^\dagger b_m^\dagger\,|0\rangle\left[\epsilon_{ijpqr}\mathsf{H}^{(\mathsf{560})pqr}_{xy}\right.\\
&\quad \left. +\frac{1}{48}\left(\epsilon_{ijpqy}\mathsf{H}^{(\mathsf{560})pq}_{x} - \epsilon_{ijpqx}\mathsf{H}^{(\mathsf{560})pq}_{y}\right) - \frac{1}{144}\epsilon_{ijpxy}\mathsf{H}^{(\mathsf{560})p}\right] + b_i^\dagger\,|0\rangle\,\mathsf{H}^{(\mathsf{560})i}_{xy}, \qquad \text{(A.15)}\\
|\overline{\Theta}^{(\mathsf{560})}_{c_x \bar c_y}\rangle &= b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger\,|0\rangle\left[\mathsf{H}^{(\mathsf{560})x}_{y} + \frac{1}{5}\delta_y^x\mathsf{H}^{(\mathsf{560})}\right] + \frac{1}{12}\epsilon^{ijklm} b_k^\dagger b_l^\dagger b_m^\dagger\,|0\rangle\left[\frac{1}{4}\left(\delta_i^x\mathsf{H}^{(\mathsf{560})}_{jy} - \delta_j^x\mathsf{H}^{(\mathsf{560})}_{iy}\right)\right.\\
&\quad \left. -\frac{1}{72}\left(4\delta_y^x\widehat{\mathsf{H}}^{(\mathsf{560})}_{ij} - \delta_j^x\widehat{\mathsf{H}}^{(\mathsf{560})}_{iy} + \delta_i^x\widehat{\mathsf{H}}^{(\mathsf{560})}_{jy}\right) - \frac{1}{6}\mathsf{H}^{(\mathsf{560})x}_{ijy} + \epsilon_{ijpqr}\mathsf{H}^{(\mathsf{560})[pqr]x}_{y}\right]\\
&\quad +b_i^\dagger\,|0\rangle\left[\frac{1}{24}\left(5\delta_y^x\mathsf{H}^{(\mathsf{560})i} - \delta_y^i\mathsf{H}^{(\mathsf{560})x}\right) - \frac{1}{2}\left(\mathsf{H}^{(\mathsf{560})ix}_{y} + \mathsf{H}^{(\mathsf{560})ix}_{(S)y}\right)\right], \qquad \text{(A.16)}
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{\mathsf{560}}\left[\overline{\Theta}^{(\mathsf{560})}_{\mu\nu}\right] &= \overline{\mathsf{1}}(5)\left[\mathsf{H}^{(\mathsf{560})}\right] + \mathsf{5}(-3)\left[\mathsf{H}^{(\mathsf{560})i}\right] + \mathsf{10}(9)\left[\mathsf{H}^{(\mathsf{560})ij}\right] + \overline{\mathsf{10}}(1)\left[\mathsf{H}^{(\mathsf{560})}_{ij}\right]\\
&\quad +\overline{\mathsf{10}}(1)\left[\widehat{\mathsf{H}}^{(\mathsf{560})}_{ij}\right] + \overline{\mathsf{24}}(5)\left[\mathsf{H}^{(\mathsf{560})i}_{j}\right] + \overline{\mathsf{40}}(1)\left[\mathsf{H}^{(\mathsf{560})l}_{ijk}\right] + \overline{\mathsf{45}}(-7)\left[\mathsf{H}^{(\mathsf{560})k}_{ij}\right]\\
&\quad +\mathsf{45}(-3)\left[\mathsf{H}^{(\mathsf{560})ij}_{k}\right] + \mathsf{50}(-3)\left[\mathsf{H}^{(\mathsf{560})ijk}_{lm}\right] + \mathsf{70}(-3)\left[\mathsf{H}^{(\mathsf{560})ij}_{(S)k}\right]\\
&\quad +\overline{\mathsf{75}}(5)\left[\mathsf{H}^{(\mathsf{560})ij}_{kl}\right] + \overline{\mathsf{175}}(1)\left[\mathsf{H}^{(\mathsf{560})[ijk]l}_{m}\right]. \qquad \text{(A.17)}
\end{aligned}
$$

Eqs. (A.11)–(A.16) show how the various $\mathsf{SU}(5)$ components given in eqs. (A.1) and (A.17) enter in the oscillator decomposition of the constrained $\mathsf{560} + \overline{\mathsf{560}}$ multiplet.

A.2 Normalization of kinetic energies of irreducible $\mathsf{SU(5)}$ multiplets arising in the decomposition of $\mathbf{560} + \overline{\mathbf{560}}$ multiplets

We exhibit below the decomposition of the kinetic energy of the $\mathsf{560}-$plet in terms of the kinetic energies of $\mathsf{SU(5)}$ multiplets residing in $\mathsf{560}$.

$$
\begin{aligned}
\mathcal{L}_{\rm KE}^{(560)} &= -\langle \partial_A \Theta_{\mu\nu}^{(560)} | \partial^A \Theta_{\mu\nu}^{(560)} \rangle \\
&= -\partial_A X_{\mu\nu}^{\dagger} \partial^A X_{\mu\nu} - \frac{1}{2} \partial_A X_{ij\mu\nu}^{\dagger} \partial^A X_{\mu\nu}^{ij} - \partial_A X_{\mu\nu}^{i\dagger} \partial^A X_{i\mu\nu} \\
&= -\partial_A X_{c_i c_j}^{\dagger} \partial^A X_{c_i c_j} - \partial_A X_{\bar{c}_i \bar{c}_j}^{\dagger} \partial^A X_{\bar{c}_i \bar{c}_j} - 2\partial_A X_{c_i \bar{c}_j}^{\dagger} \partial^A X_{c_i \bar{c}_j} \\
&\quad -\frac{1}{2} \partial_A X_{ij c_k c_l}^{\dagger} \partial^A X_{c_k c_l}^{ij} - \frac{1}{2} \partial_A X_{ij \bar{c}_k \bar{c}_l}^{\dagger} \partial^A X_{\bar{c}_k \bar{c}_l}^{ij} - \partial_A X_{ij c_k \bar{c}_l}^{\dagger} \partial^A X_{c_k \bar{c}_l}^{ij} \\
&\quad -\partial_A X_{c_j c_k}^{i\dagger} \partial^A X_{i c_j c_k} - \partial_A X_{\bar{c}_j \bar{c}_k}^{i\dagger} \partial^A X_{i \bar{c}_j \bar{c}_k} - 2\partial_A X_{c_j \bar{c}_k}^{i\dagger} \partial^A X_{i c_j \bar{c}_k} \\
&= -\frac{9}{20} \partial_A \mathsf{H}^{(560)\dagger} \partial^A \mathsf{H}^{(560)} - \frac{721}{1728} \partial_A \mathsf{H}^{(560)i\dagger} \partial^A \mathsf{H}_i^{(560)} - \partial_A \mathsf{H}^{(560)ij\dagger} \partial^A \mathsf{H}_{ij}^{(560)} \\
&\quad -\frac{3}{2} \partial_A \mathsf{H}_{ij}^{(560)\dagger} \partial^A \mathsf{H}^{(560)ij} - \frac{865}{72} \partial_A \widehat{\mathsf{H}}_{ij}^{(560)\dagger} \partial^A \widehat{\mathsf{H}}^{(560)ij} - \frac{8}{3} \partial_A \mathsf{H}_j^{(560)i\dagger} \partial^A \mathsf{H}_i^{(560)j} \\
&\quad -\frac{433}{36} \partial_A \mathsf{H}_{ijk}^{(560)l\dagger} \partial^A \mathsf{H}_l^{(560)ijk} - \partial_A \mathsf{H}_{jk}^{(560)i\dagger} \partial^A \mathsf{H}_i^{(560)jk} - \frac{145}{288} \partial_A \mathsf{H}_k^{(560)ij\dagger} \partial^A \mathsf{H}_{ij}^{(560)k} \\
&\quad -6\partial_A \mathsf{H}_{lm}^{(560)ijk\dagger} \partial^A \mathsf{H}_{ijk}^{(560)lm} - \frac{1}{2} \partial_A \mathsf{H}_{(S)k}^{(560)ij\dagger} \partial^A \mathsf{H}_{(S)ij}^{(560)k} - \frac{1}{2} \partial_A \mathsf{H}_{ij}^{(560)kl\dagger} \partial^A \mathsf{H}_{kl}^{(560)ij} \\
&\quad -12\partial_A \mathsf{H}_m^{(560)[ijk]l\dagger} \partial^A \mathsf{H}_{[ijk]l}^{(560)m},
\end{aligned}
$$

where $A = 0, \cdots 3$ is the Dirac index. In terms of normalized $\mathsf{SU(5)}$ fields, the kinetic energy of $\mathsf{560}$ tensor-spinor takes the form

$$
\begin{aligned}
\mathcal{L}_{\rm KE}^{(560)} =& -\partial_A \mathbf{H}^{(1_{560})\dagger} \partial^A \mathbf{H}^{(1_{560})} - \partial_A \mathbf{H}^{(\overline{5}_{560})i\dagger} \partial^A \mathbf{H}_i^{(\overline{5}_{560})} - \frac{1}{2!} \partial_A \mathbf{H}^{(\overline{10}_{560})ij\dagger} \partial^A \mathbf{H}_{ij}^{(\overline{10}_{560})} \\
& -\frac{1}{2!} \partial_A \mathbf{H}_{ij}^{(10_{560})\dagger} \partial^A \mathbf{H}^{(10_{560})ij} - \frac{1}{2!} \partial_A \widehat{\mathbf{H}}_{ij}^{(10_{560})\dagger} \partial^A \widehat{\mathbf{H}}^{(10_{560})ij} - \partial_A \mathbf{H}_j^{(24_{560})i\dagger} \partial^A \mathbf{H}_i^{(24_{560})j} \\
& -\frac{1}{3!} \partial_A \mathbf{H}_{ijk}^{(40_{560})l\dagger} \partial^A \mathbf{H}_l^{(40_{560})ijk} - \frac{1}{2!} \partial_A \mathbf{H}_{jk}^{(45_{560})i\dagger} \partial^A \mathbf{H}_i^{(45_{560})jk} - \frac{1}{2!} \partial_A \mathbf{H}_k^{(\overline{45}_{560})ij\dagger} \partial^A \mathbf{H}_{ij}^{(\overline{45}_{560})k} \\
& -\frac{1}{2!} \frac{1}{3!} \partial_A \mathbf{H}_{lm}^{(\overline{50}_{560})ijk\dagger} \partial^A \mathbf{H}_{ijk}^{(\overline{50}_{560})lm} - \frac{1}{2!} \partial_A \mathbf{H}_k^{(70_{560})ij\dagger} \partial^A \mathbf{H}_{ij}^{(_{560})k} \\
& -\frac{1}{2!} \frac{1}{2!} \partial_A \mathbf{H}_{ij}^{(75_{560})kl\dagger} \partial^A \mathbf{H}_{kl}^{(75_{560})ij} - \frac{1}{3!} \partial_A \mathbf{H}_m^{(175_{560})[ijk]l\dagger} \partial^A \mathbf{H}_{[ijk]l}^{(175_{560})m}.
\end{aligned}
\tag{A.18}
$$

In a similar fashion, one can now normalize the $\mathsf{SU(5)}$ tensors in the $\overline{\mathsf{560}}$-plet. Thus together for the $\mathsf{560}$ and $\overline{\mathsf{560}}$ multiplets, we have the following relationship among the un-normalized fields H and normalized fields $\mathbf{H}$ as follows

$$
\begin{aligned}
&\begin{pmatrix}\mathsf{H}^{(560)}\\ \mathsf{H}^{(\overline{560})}\end{pmatrix}=\frac{2}{3}\sqrt{5}\begin{pmatrix}\mathbf{H}^{(1_{560})}\\ \mathbf{H}^{(\overline{1}_{\overline{560}})}\end{pmatrix}, &&
\begin{pmatrix}\mathsf{H}_i^{(560)}\\ \mathsf{H}^{(\overline{560})i}\end{pmatrix}=24\sqrt{\frac{3}{721}}\begin{pmatrix}\mathbf{H}_i^{(\overline{5}_{560})}\\ \mathbf{H}^{(5_{\overline{560}})i}\end{pmatrix},\\
&\begin{pmatrix}\mathsf{H}_{ij}^{(560)}\\ \mathsf{H}^{(\overline{560})ij}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}\mathbf{H}_{ij}^{(\overline{10}_{560})}\\ \mathbf{H}^{(10_{\overline{560}})ij}\end{pmatrix}, &&
\begin{pmatrix}\mathsf{H}^{(560)ij}\\ \mathsf{H}_{ij}^{(\overline{560})}\end{pmatrix}=\frac{1}{\sqrt{3}}\begin{pmatrix}\mathbf{H}^{(10_{560})ij}\\ \mathbf{H}_{ij}^{(\overline{10}_{\overline{560}})}\end{pmatrix},\\
&\begin{pmatrix}\widehat{\mathsf{H}}^{(560)ij}\\ \widehat{\mathsf{H}}_{ij}^{(\overline{560})}\end{pmatrix}=\frac{6}{\sqrt{865}}\begin{pmatrix}\widehat{\mathbf{H}}^{(10_{560})ij}\\ \widehat{\mathbf{H}}_{ij}^{(\overline{10}_{\overline{560}})}\end{pmatrix}, &&
\begin{pmatrix}\mathsf{H}_j^{(560)i}\\ \mathsf{H}_j^{(\overline{560})i}\end{pmatrix}=\frac{1}{2}\sqrt{\frac{3}{2}}\begin{pmatrix}\mathbf{H}_j^{(24_{560})i}\\ \mathbf{H}_j^{(24_{\overline{560}})i}\end{pmatrix},\\
&\begin{pmatrix}\mathsf{H}_l^{(560)ijk}\\ \mathsf{H}_{ijk}^{(\overline{560})l}\end{pmatrix}=\sqrt{\frac{6}{433}}\begin{pmatrix}\mathbf{H}_l^{(40_{560})ijk}\\ \mathbf{H}_{ijk}^{(\overline{40}_{\overline{560}})l}\end{pmatrix}, &&
\begin{pmatrix}\mathsf{H}_k^{(560)ij}\\ \mathsf{H}_{ij}^{(\overline{560})k}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}\mathbf{H}_k^{(45_{560})ij}\\ \mathbf{H}_{ij}^{(\overline{45}_{\overline{560}})k}\end{pmatrix},\\
&\begin{pmatrix}\mathsf{H}_{ij}^{(560)k}\\ \mathsf{H}_k^{(\overline{560})ij}\end{pmatrix}=\frac{12}{\sqrt{145}}\begin{pmatrix}\mathbf{H}_{ij}^{(\overline{45}_{560})k}\\ \mathbf{H}_k^{(45_{\overline{560}})ij}\end{pmatrix}, &&
\begin{pmatrix}\mathsf{H}_{ijk}^{(560)lm}\\ \mathsf{H}_{lm}^{(\overline{560})ijk}\end{pmatrix}=\frac{1}{6\sqrt{2}}\begin{pmatrix}\mathbf{H}_{ijk}^{(\overline{50}_{560})lm}\\ \mathbf{H}_{lm}^{(50_{\overline{560}})ijk}\end{pmatrix},\\
&\begin{pmatrix}\mathsf{H}_{kl}^{(560)ij}\\ \mathsf{H}_{kl}^{(\overline{560})ij}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}\mathbf{H}_{kl}^{(75_{560})ij}\\ \mathbf{H}_{kl}^{(75_{\overline{560}})ij}\end{pmatrix}, &&
\begin{pmatrix}\mathsf{H}_{[ijk]l}^{(560)m}\\ \mathsf{H}_m^{(\overline{560})[ijk]l}\end{pmatrix}=\frac{1}{6\sqrt{2}}\begin{pmatrix}\mathbf{H}_{[ijk]l}^{(175_{560})m}\\ \mathbf{H}_m^{(\overline{175}_{\overline{560}})[ijk]l}\end{pmatrix},\\
&\begin{pmatrix}\mathsf{H}_{(S)ij}^{(560)k}\\ \mathsf{H}_{(S)k}^{(\overline{560})ij}\end{pmatrix}=\begin{pmatrix}\mathbf{H}_{ij}^{(70_{560})k}\\ \mathbf{H}_k^{(\overline{70}_{\overline{560}})jk}\end{pmatrix}.
\end{aligned}
\tag{A.19}
$$

B Couplings of the SU(5) components 1, 24, 75 that enter in spontaneous breaking of SO(10) GUT symmetry

GUT symmetry breaking is triggered by the superpotential (see eq. (3.2)):

$$
W_{\rm GUT}=\frac{1}{2!}M_{45}\Phi_{\mu\nu}^{(45)}\Phi_{\mu\nu}^{(45)}+\lambda_{45}\,\langle\Theta_{\mu\nu}^{(560)*}|B|\overline{\Theta}_{\nu\sigma}^{(560)}\rangle\,\Phi_{\mu\sigma}^{(45)}+M_{560}\,\langle\Theta_{\mu\nu}^{(560)*}|B|\overline{\Theta}_{\mu\nu}^{(560)}\rangle \tag{B.1}
$$

Here few remarks about the second term appearing in eq. (B.1) are in order. First note that there are seemingly other viable interactions of $560+\overline{560}$ with the 45 multiplet that can appear in eq. (B.1) such as

- $\lambda'_{45}\,\langle\Theta_{\mu\nu}^{(560)*}|B\Gamma_\rho\Gamma_\sigma|\overline{\Theta}_{\rho\sigma}^{(560)}\rangle\,\Phi_{\mu\nu}^{(45)}$,
- $\lambda''_{45}\,\langle\Theta_{\mu\nu}^{(560)*}|B\Gamma_\nu\Gamma_\rho|\overline{\Theta}_{\rho\sigma}^{(560)}\rangle\,\Phi_{\mu\sigma}^{(45)}$, etc.
- $\lambda'''_{45}\;\epsilon_{\mu_1\mu_1\ldots\mu_9\mu_{10}}\,\langle\Theta_{\mu_1\mu_2}^{(560)*}|B\Gamma_{\mu_3}\Gamma_{\mu_4}\Gamma_{\mu_5}\Gamma_{\mu_6}|\overline{\Theta}_{\mu_7\mu_8}^{(560)}\rangle\,\Phi_{\mu_9\mu_{10}}^{(45)}$

The first two couplings are zero. This is due to the constraint $\Gamma_\alpha\,|\overline{\Theta}_{\alpha\beta}^{(560)}\rangle=0$. The last term above is non-vanishing but we would not take it into account and in this analysis we consider only the simplest coupling of $560+\overline{560}$ with the 45 as given in eq. (B.1).

Further, the second term appearing in eq. (B.1) should actually read $\frac{\lambda_{45}}{2!}(\langle\Theta_{\mu\nu}^{(560)*}|B|\overline{\Theta}_{\nu\sigma}^{(560)}\rangle - \langle\Theta_{\sigma\nu}^{(560)*}|B|\overline{\Theta}_{\nu\mu}^{(560)}\rangle)\Phi_{\mu\sigma}^{(45)}$. This is due to the fact that only the piece antisymmetric in indices (μ,σ) of the product $\langle\Theta_{\mu\nu}^{(560)*}|B|\overline{\Theta}_{\nu\sigma}^{(560)}\rangle$ will couple to the antisymmetric tensor $\Phi_{\mu\sigma}^{(45)}$.

Eliminating the 45 multiplet through $\frac{\partial W_{\rm GUT}}{\partial \Phi^{45}_{\alpha\beta}} = 0$, we get $\Phi^{(45)}_{\alpha\beta} = -\frac{\lambda_{45}}{2M_{45}}(\langle\Theta^{(560)*}_{\alpha\nu}|B|\overline{\Theta}^{(\overline{560})}_{\nu\beta}\rangle - \langle\Theta^{(560)*}_{\beta\nu}|B|\overline{\Theta}^{(\overline{560})}_{\nu\alpha}\rangle)$, which correctly exhibits antisymmetry in indices (α,β). Finally,

$$
\begin{aligned}
W_{\rm GUT}&\\
&= -\frac{\lambda_{45}^2}{4M_{45}}\left[\langle\Theta^{(560)*}_{\alpha\nu}|B|\overline{\Theta}^{(\overline{560})}_{\nu\beta}\rangle\,\langle\Theta^{(560)*}_{\alpha\mu}|B|\overline{\Theta}^{(\overline{560})}_{\mu\beta}\rangle - \langle\Theta^{(560)*}_{\alpha\nu}|B|\overline{\Theta}^{(\overline{560})}_{\nu\beta}\rangle\,\langle\Theta^{(560)*}_{\beta\mu}|B|\overline{\Theta}^{(\overline{560})}_{\mu\alpha}\rangle\right]\\
&\quad + M_{560}\,\langle\Theta^{(560)*}_{\mu\nu}|B|\overline{\Theta}^{(\overline{560})}_{\mu\nu}\rangle \qquad\qquad (B.2)\\
&= \frac{\lambda_{45}^2}{4M_{45}}\Big[-2\,\langle\Theta^{(560)*}_{c_i\bar c_k}|B|\overline{\Theta}^{(\overline{560})}_{c_kc_j}\rangle\,\langle\Theta^{(560)*}_{\bar c_i\bar c_l}|B|\overline{\Theta}^{(\overline{560})}_{c_l\bar c_j}\rangle - 2\,\langle\Theta^{(560)*}_{c_i\bar c_k}|B|\overline{\Theta}^{(\overline{560})}_{c_k\bar c_j}\rangle\,\langle\Theta^{(560)*}_{c_l\bar c_i}|B|\overline{\Theta}^{(\overline{560})}_{c_j\bar c_l}\rangle\\
&\qquad - 2\,\langle\Theta^{(560)*}_{c_i\bar c_k}|B|\overline{\Theta}^{(\overline{560})}_{c_k\bar c_j}\rangle\,\langle\Theta^{(560)*}_{\bar c_i\bar c_l}|B|\overline{\Theta}^{(\overline{560})}_{c_lc_j}\rangle + 2\,\langle\Theta^{(560)*}_{c_i\bar c_k}|B|\overline{\Theta}^{(\overline{560})}_{c_kc_j}\rangle\,\langle\Theta^{(560)*}_{\bar c_j\bar c_l}|B|\overline{\Theta}^{(\overline{560})}_{c_l\bar c_i}\rangle\\
&\qquad + 2\,\langle\Theta^{(560)*}_{c_k\bar c_i}|B|\overline{\Theta}^{(\overline{560})}_{c_j\bar c_k}\rangle\,\langle\Theta^{(560)*}_{\bar c_j\bar c_l}|B|\overline{\Theta}^{(\overline{560})}_{c_lc_i}\rangle + \langle\Theta^{(560)*}_{c_i\bar c_k}|B|\overline{\Theta}^{(\overline{560})}_{c_k\bar c_j}\rangle\,\langle\Theta^{(560)*}_{c_j\bar c_l}|B|\overline{\Theta}^{(\overline{560})}_{c_l\bar c_i}\rangle\\
&\qquad + \langle\Theta^{(560)*}_{c_k\bar c_i}|B|\overline{\Theta}^{(\overline{560})}_{c_j\bar c_k}\rangle\,\langle\Theta^{(560)*}_{c_l\bar c_j}|B|\overline{\Theta}^{(\overline{560})}_{c_i\bar c_l}\rangle + \langle\Theta^{(560)*}_{\bar c_i\bar c_k}|B|\overline{\Theta}^{(\overline{560})}_{c_kc_j}\rangle\,\langle\Theta^{(560)*}_{\bar c_j\bar c_l}|B|\overline{\Theta}^{(\overline{560})}_{c_lc_i}\rangle + \ldots\Big]\\
&\quad - M_{560}\left[\langle\Theta^{(560)*}_{\bar c_i\bar c_j}|B|\overline{\Theta}^{(\overline{560})}_{c_jc_i}\rangle + 2\,\langle\Theta^{(560)*}_{c_i\bar c_j}|B|\overline{\Theta}^{(\overline{560})}_{c_j\bar c_i}\rangle + \ldots\right] \qquad\qquad (B.3)
\end{aligned}
$$

Using eqs. (A.11)–(A.16) and eq. (A.19) in eq. (B.3) we get

$$
\begin{aligned}
&W_{\rm GUT}\\
&= \frac{\lambda_{45}^2}{4M_{45}}\Big[-\frac{1}{16}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zj}\mathbf{H}_{jr}^{(75_{560})pq}\mathbf{H}_{pq}^{(\overline{75}_{\overline{560}})ri}\\
&-\frac{1}{8\sqrt{3}}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zj}\mathbf{H}_{jp}^{(75_{560})iq}\mathbf{H}_{q}^{(\overline{24}_{\overline{560}})p} - \frac{1}{8\sqrt{3}}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zj}\mathbf{H}_{q}^{(24_{560})p}\mathbf{H}_{pj}^{(\overline{75}_{\overline{560}})qi}\\
&-\frac{5}{24}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zj}\mathbf{H}_{p}^{(24_{560})i}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})p} + \frac{11}{48}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zj}\mathbf{H}_{j}^{(24_{560})p}\mathbf{H}_{p}^{(\overline{24}_{\overline{560}})i}\\
&+\frac{1}{48}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zi}\mathbf{H}_{q}^{(24_{560})p}\mathbf{H}_{p}^{(\overline{24}_{\overline{560}})q} - \frac{1}{48}\mathbf{H}_{iy}^{(75_{560})jx}\mathbf{H}_{x}^{(\overline{24}_{\overline{560}})y}\mathbf{H}_{jq}^{(75_{560})ip}\mathbf{H}_{p}^{(\overline{24}_{\overline{560}})q}\\
&-\frac{1}{48}\mathbf{H}_{y}^{(24_{560})x}\mathbf{H}_{xi}^{(\overline{75}_{\overline{560}})yj}\mathbf{H}_{q}^{(24_{560})p}\mathbf{H}_{pj}^{(\overline{75}_{\overline{560}})qi} - \frac{1}{24}\mathbf{H}_{iy}^{(75_{560})jx}\mathbf{H}_{x}^{(\overline{24}_{\overline{560}})y}\mathbf{H}_{q}^{(24_{560})p}\mathbf{H}_{pj}^{(\overline{75}_{\overline{560}})qi}\\
&+\frac{1}{8\sqrt{30}}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zj}\mathbf{H}_{j}^{(24_{560})i}\mathbf{H}^{(\overline{1}_{\overline{560}})} + \frac{1}{8\sqrt{30}}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zj}\mathbf{H}^{(1_{560})}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})i}\\
&+\frac{1}{90}\mathbf{H}_{iz}^{(75_{560})xy}\mathbf{H}_{xy}^{(\overline{75}_{\overline{560}})zi}\mathbf{H}^{(1_{560})}\mathbf{H}^{(\overline{1}_{\overline{560}})}\\
&+\frac{1}{48\sqrt{3}}\mathbf{H}_{iy}^{(75_{560})jx}\mathbf{H}_{x}^{(\overline{24}_{\overline{560}})y}\mathbf{H}_{p}^{(24_{560})i}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})p} + \frac{1}{48\sqrt{3}}\mathbf{H}_{y}^{(24_{560})x}\mathbf{H}_{xi}^{(\overline{75}_{\overline{560}})yj}\mathbf{H}_{p}^{(24_{560})i}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})p}\\
&+\frac{1}{24\sqrt{10}}\mathbf{H}_{iy}^{(75_{560})jx}\mathbf{H}_{x}^{(\overline{24}_{\overline{560}})y}\mathbf{H}_{j}^{(24_{560})i}\mathbf{H}^{(\overline{1}_{\overline{560}})} + \frac{1}{24\sqrt{10}}\mathbf{H}_{iy}^{(75_{560})jx}\mathbf{H}_{x}^{(\overline{24}_{\overline{560}})y}\mathbf{H}^{(1_{560})}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})i}\\
&+\frac{1}{24\sqrt{10}}\mathbf{H}_{y}^{(24_{560})x}\mathbf{H}_{xi}^{(\overline{75}_{\overline{560}})yj}\mathbf{H}_{j}^{(24_{560})i}\mathbf{H}^{(\overline{1}_{\overline{560}})} + \frac{1}{24\sqrt{10}}\mathbf{H}_{y}^{(24_{560})x}\mathbf{H}_{xi}^{(\overline{75}_{\overline{560}})yj}\mathbf{H}^{(1_{560})}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})i}\\
&-\frac{221}{576}\mathbf{H}_{x}^{(24_{560})j}\mathbf{H}_{i}^{(\overline{24}_{\overline{560}})x}\mathbf{H}_{y}^{(24_{560})i}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})y} + \frac{55}{144}\mathbf{H}_{x}^{(24_{560})j}\mathbf{H}_{i}^{(\overline{24}_{\overline{560}})x}\mathbf{H}_{j}^{(24_{560})y}\mathbf{H}_{y}^{(\overline{24}_{\overline{560}})i}\\
&-\frac{7}{576}\mathbf{H}_{y}^{(24_{560})x}\mathbf{H}_{x}^{(\overline{24}_{\overline{560}})y}\mathbf{H}_{j}^{(24_{560})i}\mathbf{H}_{i}^{(\overline{24}_{\overline{560}})j}\\
&-\frac{1}{48\sqrt{30}}\mathbf{H}_{x}^{(24_{560})i}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})x}\mathbf{H}_{i}^{(24_{560})j}\mathbf{H}^{(\overline{1}_{\overline{560}})} - \frac{1}{48\sqrt{30}}\mathbf{H}_{x}^{(24_{560})i}\mathbf{H}_{j}^{(\overline{24}_{\overline{560}})x}\mathbf{H}^{(1_{560})}\mathbf{H}_{i}^{(\overline{24}_{\overline{560}})j}
\end{aligned}
$$

$$-\frac{1}{480}\mathbf{H}_i^{(24_{560})j}\mathbf{H}^{(\overline{1}_{\overline{560}})}\mathbf{H}_j^{(24_{560})i}\mathbf{H}^{(\overline{1}_{\overline{560}})} - \frac{1}{480}\mathbf{H}^{(1_{560})}\mathbf{H}_i^{(\overline{24}_{\overline{560}})j}\mathbf{H}^{(1_{560})}\mathbf{H}_j^{(\overline{24}_{\overline{560}})i}$$

$$+\frac{1}{144}\mathbf{H}_i^{(24_{560})j}\mathbf{H}^{(\overline{1}_{\overline{560}})}\mathbf{H}^{(1_{560})}\mathbf{H}_j^{(\overline{24}_{\overline{560}})i} - \frac{1}{405}\mathbf{H}^{(1_{560})}\mathbf{H}^{(\overline{1}_{\overline{560}})}\mathbf{H}^{(1_{560})}\mathbf{H}^{(\overline{1}_{\overline{560}})} + \ldots\Bigg]$$

$$+ iM_{560}\left[\frac{1}{4}\mathbf{H}_{kl}^{(75_{560})ij}\mathbf{H}_{ij}^{(\overline{75}_{\overline{560}})kl} + \mathbf{H}_j^{(24_{560})i}\mathbf{H}_i^{(\overline{24}_{\overline{560}})j} + \mathbf{H}^{(1_{560})}\mathbf{H}^{(\overline{1}_{\overline{560}})} + \ldots\right] \tag{B.4}$$

The ellipsis above denote terms containing $\mathsf{SU}(5)$ fields other than $\mathsf{75}(-5)$, $\mathsf{24}(-5)$ and $\mathsf{1}(-5)$. Those unrepresented fields do not enter into GUT Symmetry breaking.

We identify the $\mathsf{SU}(3)_\mathsf{C} \times \mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y}$ singlets through

$$\begin{pmatrix}\mathbf{H}^{(1_{560})}\\ \mathbf{H}^{(\overline{1}_{\overline{560}})}\end{pmatrix} \equiv \begin{pmatrix}\mathcal{S}_1\\ \overline{\mathcal{S}}_1\end{pmatrix}, \qquad \begin{pmatrix}\mathbf{H}_\beta^{(24_{560})\alpha}\\ \mathbf{H}_\beta^{(\overline{24}_{\overline{560}})\alpha}\end{pmatrix} = \frac{1}{3}\delta_\beta^\alpha\begin{pmatrix}\mathcal{S}_{24}\\ \overline{\mathcal{S}}_{24}\end{pmatrix} + \ldots,$$

$$\begin{pmatrix}\mathbf{H}_b^{(24_{560})a}\\ \mathbf{H}_b^{(\overline{24}_{\overline{560}})a}\end{pmatrix} = -\frac{1}{2}\delta_b^a\begin{pmatrix}\mathcal{S}_{24}\\ \overline{\mathcal{S}}_{24}\end{pmatrix} + \ldots, \qquad \begin{pmatrix}\mathbf{H}_{\gamma\sigma}^{(75_{560})\alpha\beta}\\ \mathbf{H}_{\gamma\sigma}^{(\overline{75}_{\overline{560}})\alpha\beta}\end{pmatrix} = \frac{1}{6}\left(\delta_\gamma^\alpha\delta_\sigma^\beta - \delta_\sigma^\alpha\delta_\gamma^\beta\right)\begin{pmatrix}\mathcal{S}_{75}\\ \overline{\mathcal{S}}_{75}\end{pmatrix} + \ldots,$$

$$\begin{pmatrix}\mathbf{H}_{cd}^{(75_{560})ab}\\ \mathbf{H}_{cd}^{(\overline{75}_{\overline{560}})ab}\end{pmatrix} = \frac{1}{2}\left(\delta_c^a\delta_d^b - \delta_d^a\delta_c^b\right)\begin{pmatrix}\mathcal{S}_{75}\\ \overline{\mathcal{S}}_{75}\end{pmatrix}, \qquad \begin{pmatrix}\mathbf{H}_{\beta b}^{(75_{560})\alpha a}\\ \mathbf{H}_{\beta b}^{(\overline{75}_{\overline{560}})\alpha a}\end{pmatrix} = -\frac{1}{6}\delta_b^a\delta_\beta^\alpha\begin{pmatrix}\mathcal{S}_{75}\\ \overline{\mathcal{S}}_{75}\end{pmatrix} + \ldots, \tag{B.5}$$

where $\alpha,\ \beta,\ \gamma,\ \sigma = 1,\ 2,\ 3$ are $\mathsf{SU}(3)$ color indices and $a,\ b,\ c,\ d = 4,\ 5$ are $\mathsf{SU}(2)$ weak indices. The ellipsis above represent terms containing fields which are not $\mathsf{SU}(3)_\mathsf{C} \times \mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y}$ singlets $(1,1)(0)$. The above singlets are normalized according to [18]

$$\begin{pmatrix}\mathcal{S}_1\\ \overline{\mathcal{S}}_1\end{pmatrix} = \begin{pmatrix}\mathbf{S}_1\\ \overline{\mathbf{S}}_1\end{pmatrix}, \qquad \begin{pmatrix}\mathcal{S}_{24}\\ \overline{\mathcal{S}}_{24}\end{pmatrix} = \sqrt{\frac{6}{5}}\begin{pmatrix}\mathbf{S}_{24}\\ \overline{\mathbf{S}}_{24}\end{pmatrix}, \qquad \begin{pmatrix}\mathcal{S}_{75}\\ \overline{\mathcal{S}}_{75}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\mathbf{S}_{75}\\ \overline{\mathbf{S}}_{75}\end{pmatrix}. \tag{B.6}$$

Finally, we denote the VEVs of the normalized fields as follows

$$\begin{pmatrix}\mathcal{V}_1\\ \overline{\mathcal{V}}_1\end{pmatrix} \equiv \begin{pmatrix}\langle\mathbf{S}_1\rangle\\ \langle\overline{\mathbf{S}}_1\rangle\end{pmatrix}, \qquad \begin{pmatrix}\mathcal{V}_{24}\\ \overline{\mathcal{V}}_{24}\end{pmatrix} \equiv \begin{pmatrix}\langle\mathbf{S}_{24}\rangle\\ \langle\overline{\mathbf{S}}_{24}\rangle\end{pmatrix}, \qquad \begin{pmatrix}\mathcal{V}_{75}\\ \overline{\mathcal{V}}_{75}\end{pmatrix} \equiv \begin{pmatrix}\langle\mathbf{S}_{75}\rangle\\ \langle\overline{\mathbf{S}}_{75}\rangle\end{pmatrix}. \tag{B.7}$$

Inserting eqs. (B.5)–(B.7) into eq. (B.4) gives eq. (3.3).

C Extraction and normalization of $\mathsf{SU}(2)_\mathsf{L}$ doublet fields in the model

C.1 $\mathsf{5} + \overline{\mathsf{5}}$ and $\mathsf{45} + \overline{\mathsf{45}}$ of $\mathsf{SU}(5)$

The $\mathsf{5} + \overline{\mathsf{5}}$ plets of $\mathsf{SU}(5)$ are normalized according to (see [33])

$$\mathsf{H}^{(10_r)i} = \sqrt{2}\mathbf{H}^{(5_{10_r})i}, \qquad \mathsf{H}_i^{(\overline{10_r})} = \sqrt{2}\mathbf{H}_i^{(\overline{5}_{\overline{10_r}})} \tag{C.1}$$

The identification of $\mathsf{SU}(2)$ doublets, denoted by D's, contained in $\mathsf{5} + \overline{\mathsf{5}}$ and $\mathsf{45} + \overline{\mathsf{45}}$, are done through (see [18])

$$\mathbf{H}^{(5_\#)a} \equiv {}^{(5_\#)}\mathsf{D}^a; \qquad \mathbf{H}_a^{(\overline{5}_\#)} \equiv {}^{(\overline{5}_\#)}\mathsf{D}_a, \tag{C.2}$$

$$\mathbf{H}_\beta^{(45_\#)\alpha a} = \frac{1}{3}\delta_\beta^\alpha\ {}^{(45_\#)}\mathsf{D}^a + \ldots; \qquad \mathbf{H}_c^{(45_\#)ab} = \delta_c^b\ {}^{(45_\#)}\mathsf{D}^a - \delta_c^a\ {}^{(45_\#)}\mathsf{D}^b,$$

$$\mathbf{H}_{\alpha a}^{(\overline{45}_\#)\beta} = \frac{1}{3}\delta_\alpha^\beta\ {}^{(\overline{45}_\#)}\mathsf{D}_a \ldots; \qquad \mathbf{H}_{ab}^{(\overline{45}_\#)c} = \delta_b^c\ {}^{(\overline{45}_\#)}\mathsf{D}_a - \delta_a^c\ {}^{(\overline{45}_\#)}\mathsf{D}_b, \tag{C.3}$$

where $a,\ b,\ c = 4,\ 5$ are reserved for SU(2) weak indices and $\alpha,\ \beta,\ \gamma, = 1,\ 2,\ 3$ are SU(3) color indices. The ellipsis above represent terms containing fields which are not $\mathsf{SU}(3)_\mathsf{C} \times \mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y}$ doublets (1, 2)(3). The symbol # refers to the 10_r, 320, $560 + \overline{560}$ fields of SO(10). Note, however, that D's are un-normalized. To normalize the SU(2) doublets contained in the SO(10) tensors, we carry out the following field redefinition ([18]):

$$
\begin{aligned}
{}^{(5_\#)}\mathsf{D}^a &= {}^{(5_\#)}\mathbf{D}^a; &\qquad {}^{(\overline{5}_\#)}\mathsf{D}_a &= {}^{(\overline{5}_\#)}\mathbf{D}_a, \\
{}^{(45_\#)}\mathsf{D}^a &= \frac{1}{2}\sqrt{\frac{3}{2}}\ {}^{(45_\#)}\mathbf{D}^a; &\qquad {}^{(\overline{45}_\#)}\mathsf{D}_a &= \frac{1}{2}\sqrt{\frac{3}{2}}\ {}^{(\overline{45}_\#)}\mathbf{D}_a,
\end{aligned} \tag{C.4}
$$

where **D**'s represent the doublet fields with canonically normalized kinetic energy terms.

C.2 $70 + \overline{70}$ of SU(5)

The 70-plet of SU(5) has the following $\mathsf{SU}(3)_\mathsf{C} \times \mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y}$ decomposition [21]

$$
\begin{aligned}
70\left[\mathbf{H}_k^{(70_\#)ij}\right] = &\ (1,2)(3)\left[{}^{(70_\#)}\mathsf{D}^a\right] + (3,1)(-2)\left[{}^{(70_\#)}\mathsf{T}^\alpha\right] + (1,4)(3)\left[{}^{(70_\#)}\psi_c^{\{ab\}}\right] \\
&+(3,3)(-2)\left[{}^{(70_\#)}\psi_b^{a\alpha}\right] + (\overline{3},3)(8)\left[{}^{(70_\#)}\psi_\alpha^{\{ab\}}\right] + (8,2)(3)\left[{}^{(70_\#)}\psi_\beta^{\alpha a}\right] \\
&+(6,2)(-7)\left[{}^{(70_\#)}\psi_a^{\{\alpha\beta\}}\right] + (15,1)(-2)\left[{}^{(70_\#)}\psi_\gamma^{\{\alpha\beta\}}\right],
\end{aligned} \tag{C.5}
$$

where we have defined

$$
\begin{aligned}
\mathbf{H}_\alpha^{(70_\#)\alpha a} = -\mathbf{H}_b^{(70_\#)ba} &\equiv {}^{(70_\#)}\mathsf{D}^a; &\qquad \mathbf{H}_\beta^{(70_\#)\beta\alpha} = -\mathbf{H}_a^{(70_\#)a\alpha} &\equiv {}^{(70_\#)}\mathsf{T}^\alpha; \\
\mathbf{H}_\alpha^{(70_\#)ab} &\equiv {}^{(70_\#)}\psi_\alpha^{\{ab\}}; &\qquad \mathbf{H}_a^{(70_\#)\alpha\beta} &\equiv {}^{(70_\#)}\psi_a^{\{\alpha\beta\}}.
\end{aligned} \tag{C.6}
$$

Here # refers to 320 and $\overline{560}$. The first two relationships in eq. (C.6) follow from the tracelessness condition on the SU(5) irreducible tensor $\mathbf{H}_k^{(70_\#)ij}$. The reducible tensors of the 70-plet can be expressed in terms of the irreducible ones as follows:

$$
\begin{aligned}
\mathbf{H}_b^{(70_\#)a\alpha} &= {}^{(70_\#)}\psi_b^{a\alpha} - \frac{1}{2}\delta_b^a\ {}^{(70_\#)}\mathsf{T}^\alpha; \qquad \mathbf{H}_\beta^{(70_\#)\alpha a} = {}^{(70_\#)}\psi_\beta^{\alpha a} + \frac{1}{3}\delta_\beta^\alpha\ {}^{(70_\#)}\mathsf{D}^a; \\
\mathbf{H}_c^{(70_\#)ab} &= {}^{(70_\#)}\psi_c^{\{ab\}} - \frac{1}{3}\left(\delta_c^b\ {}^{(70_\#)}\mathsf{D}^a + \delta_c^a\ {}^{(70_\#)}\mathsf{D}^b\right); \\
\mathbf{H}_\gamma^{(70_\#)\alpha\beta} &= {}^{(70_\#)}\psi_\gamma^{\{\alpha\beta\}} + \frac{1}{4}\left(\delta_\gamma^\alpha\ {}^{(70_\#)}\mathsf{T}^\beta + \delta_\gamma^\beta\ {}^{(70_\#)}\mathsf{T}^\alpha\right).
\end{aligned} \tag{C.7}
$$

The kinetic energy of the 70-plet is given by

$$
\begin{aligned}
\mathcal{L}_{\mathrm{KE}}^{(70_\#)} &= -\partial_A \mathbf{H}_k^{(70_\#)ij} \partial^A \mathbf{H}_k^{(70_\#)ij\dagger} \\
&= -\Big[\partial_A {}^{(70_\#)}\psi_\gamma^{\{\alpha\beta\}}\ \partial^A {}^{(70_\#)}\psi_\gamma^{\{\alpha\beta\}\dagger} + \partial_A {}^{(70_\#)}\psi_c^{\{ab\}}\ \partial^A {}^{(70_\#)}\psi_c^{\{ab\}\dagger} \\
&\quad + \partial_A {}^{(70_\#)}\psi_\alpha^{\{ab\}}\ \partial^A {}^{(70_\#)}\psi_\alpha^{\{ab\}\dagger} + \partial_A {}^{(70_\#)}\psi_a^{\{\alpha\beta\}}\ \partial^A {}^{(70_\#)}\psi_a^{\{\alpha\beta\}\dagger} \\
&\quad + 2\ \partial_A {}^{(70_\#)}\psi_\beta^{\{\alpha a\}}\ \partial^A {}^{(70_\#)}\psi_\beta^{\{\alpha a\}\dagger} + 2\ \partial_A {}^{(70_\#)}\psi_b^{\{a\alpha\}}\ \partial^A {}^{(70_\#)}\psi_b^{\{a\alpha\}\dagger} \\
&\quad + \frac{4}{3}\ \partial^A {}^{(70_\#)}\mathsf{D}^a\ \partial_A {}^{(70_\#)}\mathsf{D}^{a\dagger} + \frac{3}{2}\ \partial^A {}^{(70_\#)}\mathsf{T}^\alpha\ \partial_A {}^{(70_\#)}\mathsf{T}^{\alpha\dagger}\Big].
\end{aligned} \tag{C.8}
$$

In terms of normalized $\mathsf{SU}(3)_\mathsf{C} \times \mathsf{SU}(2)_\mathsf{L} \times \mathsf{U}(1)_\mathsf{Y}$ fields, the kinetic energy of $\mathsf{70}$ tensor takes the form

$$\begin{aligned}\mathcal{L}_{\rm KE}^{(70_\#)} = -\Big[&\frac{1}{2!}\partial_A{}^{(70_\#)}\mathbf{\Psi}_\gamma^{\{\alpha\beta\}}\ \partial^{A(70_\#)}\mathbf{\Psi}_\gamma^{\{\alpha\beta\}\dagger} + \frac{1}{2!}\partial_A{}^{(70_\#)}\mathbf{\Psi}_c^{\{ab\}}\ \partial^{A(70_\#)}\mathbf{\Psi}_c^{\{ab\}\dagger}\\ &+\frac{1}{2!}\partial_A{}^{(70_\#)}\mathbf{\Psi}_\alpha^{\{ab\}}\ \partial^{A(70_\#)}\mathbf{\Psi}_\alpha^{\{ab\}\dagger} + \frac{1}{2!}\partial_A{}^{(70_\#)}\mathbf{\Psi}_a^{\{\alpha\beta\}}\ \partial^{A(70_\#)}\mathbf{\Psi}_a^{\{\alpha\beta\}\dagger}\\ &+\partial_A{}^{(70_\#)}\mathbf{\Psi}_\beta^{\{\alpha a\}}\ \partial^{A(70_\#)}\mathbf{\Psi}_\beta^{\{\alpha a\}\dagger} + \partial_A{}^{(70_\#)}\mathbf{\Psi}_b^{\{a\alpha\}}\ \partial^{A(70_\#)}\mathbf{\Psi}_b^{\{a\alpha\}\dagger}\\ &+\partial^{A(70_\#)}\mathbf{D}^a\ \partial_A{}^{(70_\#)}\mathbf{D}^{a\dagger} + \partial^{A(70_\#)}\mathbf{T}^\alpha\ \partial_A{}^{(70_\#)}\mathbf{T}^{\alpha\dagger}\Big]. \end{aligned} \tag{C.9}$$

One can now extend the above results to $\overline{\mathsf{70}}$ of $\mathsf{SU}(5)$ contained in $\mathsf{320}$−plet and $\mathsf{560}$−plet of $\mathsf{SO}(10)$. Therefore,

$$\begin{aligned}
&\begin{pmatrix}{}^{(70_\#)}\psi_\gamma^{\{\alpha\beta\}}\\ {}^{(\overline{70}_\#)}\psi^\gamma_{\{\alpha\beta\}}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}{}^{(70_\#)}\mathbf{\Psi}_\gamma^{\{\alpha\beta\}}\\ {}^{(\overline{70}_\#)}\mathbf{\Psi}^\gamma_{\{\alpha\beta\}}\end{pmatrix}, &&\begin{pmatrix}{}^{(70_\#)}\psi_c^{\{ab\}}\\ {}^{(\overline{70}_\#)}\psi^c_{\{ab\}}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}{}^{(70_\#)}\mathbf{\Psi}_c^{\{ab\}}\\ {}^{(\overline{70}_\#)}\mathbf{\Psi}^c_{\{ab\}}\end{pmatrix},\\
&\begin{pmatrix}{}^{(70_\#)}\psi_\alpha^{\{ab\}}\\ {}^{(\overline{70}_\#)}\psi^\alpha_{\{ab\}}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}{}^{(70_\#)}\mathbf{\Psi}_\alpha^{\{ab\}}\\ {}^{(\overline{70}_\#)}\mathbf{\Psi}^\alpha_{\{ab\}}\end{pmatrix}, &&\begin{pmatrix}{}^{(70_\#)}\psi_a^{\{\alpha\beta\}}\\ {}^{(\overline{70}_\#)}\psi^a_{\{\alpha\beta\}}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}{}^{(70_\#)}\mathbf{\Psi}_a^{\{\alpha\beta\}}\\ {}^{(\overline{70}_\#)}\mathbf{\Psi}^a_{\{\alpha\beta\}}\end{pmatrix},\\
&\begin{pmatrix}{}^{(70_\#)}\psi_\beta^{\alpha a}\\ {}^{(\overline{70}_\#)}\psi^\beta_{\alpha a}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}{}^{(70_\#)}\mathbf{\Psi}_\beta^{\alpha a}\\ {}^{(\overline{70}_\#)}\mathbf{\Psi}^\beta_{\alpha a}\end{pmatrix}, &&\begin{pmatrix}{}^{(70_\#)}\psi_b^{a\alpha}\\ {}^{(\overline{70}_\#)}\psi^b_{a\alpha}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}{}^{(70_\#)}\mathbf{\Psi}_b^{a\alpha}\\ {}^{(\overline{70}_\#)}\mathbf{\Psi}^b_{a\alpha}\end{pmatrix},\\
&\begin{pmatrix}{}^{(70_\#)}\mathsf{D}^a\\ {}^{(\overline{70}_\#)}\mathsf{D}_a\end{pmatrix} = \frac{\sqrt{3}}{2}\begin{pmatrix}{}^{(70_\#)}\mathbf{D}^a\\ {}^{(\overline{70}_\#)}\mathbf{D}_a\end{pmatrix}, &&\begin{pmatrix}{}^{(70_\#)}\mathsf{T}^\alpha\\ {}^{(\overline{70}_\#)}\mathsf{T}_\alpha\end{pmatrix} = \sqrt{\frac{2}{3}}\begin{pmatrix}{}^{(70_\#)}\mathbf{T}^\alpha\\ {}^{(\overline{70}_\#)}\mathbf{T}_\alpha\end{pmatrix}
\end{aligned} \tag{C.10}$$

D The 320 dimensional tensors of SO(10)

There are two $\mathsf{320}$ multiplets in $\mathsf{SO}(10)$ and both are three index ones and to construct these we start with the 1000 dimensional reducible multiplet $T_{\mu\nu\lambda}$. This can be decomposed as follows

$$1000\ [T_{\mu\nu\lambda}] = \mathsf{120}\ [A_{[\mu\nu\lambda]}] + \mathsf{210}'\ [S_{(\mu\nu\lambda)}] + \mathsf{320}\ [\Lambda^{(\mathsf{320}_a)}_{[\mu\nu]\lambda}] + \mathsf{320}\ [\Lambda^{(\mathsf{320}_s)}_{(\mu\nu)\lambda}] + \mathsf{10} + \mathsf{10} + \mathsf{10}$$

Here $\Lambda^{(\mathsf{320}_a)}_{[\mu\nu]\lambda}$ is the $\mathsf{320}$ multiplet which is anti-symmetric in the first two indices while $\Lambda^{(\mathsf{320}_s)}_{(\mu\nu)\lambda}$ is the tensor multiplet which is symmetric in the first two indices. We now show how the various irreducible components of $T_{\mu\nu\lambda}$ arise.

D.1 Construction of $\Lambda^{(\mathsf{320}_a)}_{[\mu\nu]\lambda}$ tensor

The three index tensor $A_{\mu\nu\lambda}$ is totally anti-symmetric in the three indices and can be constructed from $T_{\mu\nu\lambda}$ in a direct way so that

$$A_{[\mu\nu\lambda]} \equiv \frac{1}{3!}\left(T_{\mu\nu\lambda} + T_{\nu\lambda\mu} + T_{\lambda\mu\nu} - T_{\nu\mu\lambda} - T_{\lambda\nu\mu} - T_{\mu\lambda\nu}\right)$$

It has ${}^{10}C_3 = 120$ number of independent components. Next let us consider the tensor $\Lambda''_{[\mu\nu]\lambda} = \frac{1}{2}[T_{\mu\nu\lambda} - T_{\nu\mu\lambda}]$ which has 450 components and from it construct a 330 multiplet $\Lambda'_{\mu\nu\lambda}$ using $450 - 120$ which gives

$$\Lambda'{}_{[\mu\nu]\lambda} = \frac{1}{3}\left(T_{\mu\nu\lambda} - T_{\nu\mu\lambda}\right) + \frac{1}{6}\left(-T_{\nu\lambda\mu} + T_{\lambda\nu\mu} - T_{\lambda\mu\nu} + T_{\mu\lambda\nu}\right)$$

JHEP07(2025)222

Finally, we construct the irreducible ($330-10=$) $320-$dimensional tensor, $\Lambda^{(320_a)}_{[\mu\nu]\lambda}$, from $\Lambda'_{[\mu\nu]\lambda}$ by subtracting of its trace. Thus we write: $\Lambda^{(320_a)}_{[\mu\nu]\lambda}=\Lambda'_{[\mu\nu]\lambda}+\kappa\left(\delta_{\nu\lambda}\,\Lambda'_{[\mu\alpha]\alpha}-\delta_{\mu\lambda}\,\Lambda'_{[\nu\alpha]\alpha}\right)$, where $\Lambda'_{[\mu\alpha]\alpha}=\frac{1}{2}\left(T_{\mu\alpha\alpha}-T_{\alpha\mu\alpha}\right)$. Contracting this last equation with $\delta_{\nu\lambda}$ leads to $\kappa=-\frac{1}{9}$ which gives the first of the two 320 multiplets in 1000,

$$\begin{aligned}\Lambda^{(320_a)}_{[\mu\nu]\lambda}=&\frac{1}{6}\Big[2(T_{\mu\nu\lambda}-T_{\nu\mu\lambda})+T_{\lambda\nu\mu}+T_{\mu\lambda\nu}-T_{\nu\lambda\mu}-T_{\lambda\mu\nu}\Big]\\&-\frac{1}{18}\Big[\delta_{\nu\lambda}(T_{\mu\alpha\alpha}-T_{\alpha\mu\alpha})-\delta_{\mu\lambda}(T_{\nu\alpha\alpha}-T_{\alpha\nu\alpha})\Big]\end{aligned}\tag{D.1}$$

D.2 Construction of $\Lambda^{(320_s)}_{(\mu\nu)\lambda}$ tensor

Next we construct the 320 multiplet which is symmetric in the first two indices. Here the 320 arises from the 1000-plet as follows: $550-220-10$ which we now exhibit in detail. Thus from the 1000-multiplet we define a tensor symmetric in the first two indices so that

$$\widetilde{\Lambda''}_{(\mu\nu)\lambda}\equiv\frac{1}{2}\left[T_{\mu\nu\lambda}+T_{\nu\mu\lambda}\right]$$

which has a dimensionality of ${}^{11}C_2\times 10=550$. We remove from it the tensor $S_{(\mu\nu\lambda)}$ which is totally symmetric in all three indices, has a dimensionality of ${}^{12}C_3=220$ and the tensor form

$$S_{(\mu\nu\lambda)}\equiv\frac{1}{3!}\left(T_{\mu\nu\lambda}+T_{\nu\lambda\mu}+T_{\lambda\mu\nu}+T_{\nu\mu\lambda}+T_{\lambda\nu\mu}+T_{\mu\lambda\nu}\right)$$

Removing the totally symmetric part from $\widetilde{\Lambda''}_{(\mu\nu)\lambda}$ gives us the tensor $\widetilde{\Lambda'}_{(\mu\nu)\lambda}$ of dimensionality 330 ($=550-220$) where

$$\widetilde{\Lambda'}_{(\mu\nu)\lambda}=\widetilde{\Lambda''}_{(\mu\nu)\lambda}-S_{(\mu\nu\lambda)}=\frac{1}{3}\left(T_{\mu\nu\lambda}+T_{\nu\mu\lambda}\right)-\frac{1}{6}\left(T_{\nu\lambda\mu}+T_{\lambda\mu\nu}+T_{\lambda\nu\mu}+T_{\mu\lambda\nu}\right)$$

As noted the tensor $\widetilde{\Lambda'}_{(\mu\nu)\lambda}$ has 330 components and to get the second irreducible tensor 320 we need to make it traceless.

$$\Lambda^{(320_s)}_{(\mu\nu)\lambda}=\widetilde{\Lambda'}_{(\mu\nu)\lambda}+\delta_{\nu\lambda}U_\mu+\delta_{\mu\lambda}V_\nu+\delta_{\mu\nu}W_\lambda.$$

We determine U_μ, V_ν and W_λ such that $\Lambda^{(320_s)}_{(\mu\nu)\lambda}$ is traceless. Contracting the last equation with $\delta_{\nu\lambda}$, $\delta_{\mu\lambda}$, and $\delta_{\mu\nu}$ gives

$$0=\widetilde{\Lambda'}_{(\mu\alpha)\alpha}+10U_\mu+V_\mu+W_\mu,\quad 0=\widetilde{\Lambda'}_{(\alpha\nu)\alpha}+U_\nu+10V_\nu+W_\nu,\quad 0=\widetilde{\Lambda'}_{(\alpha\alpha)\lambda}+U_\lambda+V_\lambda+10W_\lambda.$$

The solution to these equations are $U_\mu=\frac{1}{54}\left(-T_{\mu\alpha\alpha}+2T_{\alpha\alpha\mu}-T_{\alpha\mu\alpha}\right)$, $V_\nu=\frac{1}{54}(-T_{\nu\alpha\alpha}+2T_{\alpha\alpha\nu}-T_{\alpha\nu\alpha})$ and $W_\lambda=\frac{1}{27}\left(T_{\lambda\alpha\alpha}-2T_{\alpha\alpha\lambda}+T_{\alpha\lambda\alpha}\right)$. Thus the second 320 plet is given by

$$\begin{aligned}\Lambda^{(320_s)}_{(\mu\nu)\lambda}=&\frac{1}{6}\Big[2(T_{\mu\nu\lambda}+T_{\nu\mu\lambda})-(T_{\lambda\nu\mu}+T_{\mu\lambda\nu}+T_{\nu\lambda\mu}+T_{\lambda\mu\nu})\Big]\\&+\frac{1}{54}\Big[\delta_{\nu\lambda}\big(-T_{\mu\alpha\alpha}+2T_{\alpha\alpha\mu}-T_{\alpha\mu\alpha}\big)+\delta_{\mu\lambda}\big(-T_{\nu\alpha\alpha}+2T_{\alpha\alpha\nu}-T_{\alpha\nu\alpha}\big)\\&-2\delta_{\mu\nu}\big(-T_{\lambda\alpha\alpha}+2T_{\alpha\alpha\lambda}-T_{\alpha\lambda\alpha}\big)\Big]\end{aligned}\tag{D.2}$$

The decomposition of the $320-$dimensional tensor under $\mathsf{SU}(5)\times\mathsf{U}(1)$ is given by

$$\begin{aligned}\mathsf{320}\left[\begin{pmatrix}\Lambda^{(320_a)}_{[\mu\nu]\lambda}\\ \Lambda^{(320_s)}_{(\mu\nu)\lambda}\end{pmatrix}\right]=&\,\mathsf{5}(2)\left[\mathsf{H}^{(320)i}\right]+\overline{\mathsf{5}}(-2)\left[\mathsf{H}^{(320)}_i\right]+\mathsf{40}(-6)\left[\mathsf{H}^{(320)ijk}_l\right]+\overline{\mathsf{40}}(6)\left[\mathsf{H}^{(320)l}_{ijk}\right]\\&+\mathsf{45}(2)\left[\mathsf{H}^{(320)ij}_k\right]+\overline{\mathsf{45}}(-2)\left[\mathsf{H}^{(320)k}_{ij}\right]+\mathsf{70}(2)\left[\mathsf{H}^{(320)ij}_{(S)k}\right]+\overline{\mathsf{70}}(-2)\left[\mathsf{H}^{(320)k}_{(S)ij}\right]\end{aligned}\tag{D.3}$$

JHEP07(2025)222

E Possible trilinear couplings of the 320-plet with the 560-plet and their generational symmetries

E.1 Couplings that involve three gammas

Consider the coupling defined through

$$J_{\grave{a}\grave{b}} = \Theta^{(560)\grave{a}\mathsf{T}}_{\alpha\beta} B\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Theta^{(560)\grave{b}}_{\alpha\beta}\ X_{\mu\nu\lambda}, \tag{E.1}$$

where for clarity, we write the expression $\langle\Theta^{(560)\grave{a}*}_{\alpha\beta}|B\Gamma_\mu\Gamma_\nu\Gamma_\lambda|\Theta^{(560)\grave{b}}_{\alpha\beta}\rangle$ as $\Theta^{(560)\grave{a}\mathsf{T}}_{\alpha\beta}B\Gamma_\mu\Gamma_\nu\Gamma_\lambda$ $\Theta^{(560)\grave{b}}_{\alpha\beta}$. Here $^\mathsf{T}$ stands for the transpose, $X_{\mu\nu\lambda}$ is a generic three-index tensor and $\grave{a}$, $\grave{b}$ are generation indices. Therefore

$$\begin{aligned}
J_{\grave{a}\grave{b}} &= \left(\Theta^{(560)\grave{a}\mathsf{T}}_{\alpha\beta}B\Gamma_\mu\Gamma_\nu\Gamma_\lambda\ \Theta^{(560)\grave{b}}_{\alpha\beta}\right)^\mathsf{T}X_{\mu\nu\lambda}\\
&= \Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}\left(\Gamma_\mu\Gamma_\nu\Gamma_\lambda\right)^\mathsf{T}\underbrace{B^\mathsf{T}}_{=-B}\left(\Theta^{(560)\grave{a}\mathsf{T}}_{\alpha\beta}\right)^\mathsf{T}X_{\mu\nu\lambda}\\
&= -\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}\Gamma^\mathsf{T}_\lambda\Gamma^\mathsf{T}_\nu\Gamma^\mathsf{T}_\mu B\Theta^{(560)\grave{a}}_{\alpha\beta}X_{\mu\nu\lambda}\\
&= -\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\underbrace{\left(B^{-1}\Gamma^\mathsf{T}_\lambda B\right)}_{=-\Gamma_\lambda}\underbrace{\left(B^{-1}\ \Gamma^\mathsf{T}_\nu B\right)}_{=-\Gamma_\nu}\underbrace{\left(B^{-1}\ \Gamma^\mathsf{T}_\mu B\right)}_{=-\Gamma_\mu}\Theta^{(560)\grave{a}}_{\alpha\beta}X_{\mu\nu\lambda}\\
&= -(-1)^3\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\Gamma_\lambda\Gamma_\nu\Gamma_\mu\Theta^{(560)\grave{a}}_{\alpha\beta}X_{\mu\nu\lambda}\\
&= \Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\qquad\underbrace{\Gamma_\lambda\Gamma_\nu\Gamma_\mu}_{2\delta_{\nu\lambda}\Gamma_\mu-2\delta_{\mu\lambda}\Gamma_\nu+\Gamma_\nu\Gamma_\mu\Gamma_\lambda\quad[\text{see }*]}\qquad\Theta^{(560)\grave{a}}_{\alpha\beta}X_{\mu\nu\lambda}\\
&= 2\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\Gamma_\mu\Theta^{(560)\grave{a}}_{\alpha\beta}X_{\mu\sigma\sigma}-2\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\Gamma_\nu\Theta^{(560)\grave{a}}_{\alpha\beta}X_{\sigma\nu\sigma}\\
&\quad+\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\Gamma_\nu\Gamma_\mu\Gamma_\lambda\Theta^{(560)\grave{a}}_{\alpha\beta}X_{\mu\nu\lambda}
\end{aligned}\tag{E.2}$$

CASE 1: $\quad X_{\mu\nu\lambda} = \Lambda^{(320_a)}_{[\mu\nu]\lambda}$

$$\begin{aligned}
J_{\grave{a}\grave{b}} &= \Theta^{(560)\grave{a}\mathsf{T}}_{\alpha\beta}B\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Theta^{(560)\grave{b}}_{\alpha\beta}\Lambda^{(320_a)}_{[\mu\nu]\lambda}\quad[\text{From eq. (E.1)}]\\
&= 2\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\Gamma_\mu\Theta^{(560)\grave{a}}_{\alpha\beta}\underbrace{\Lambda^{(320_a)}_{[\mu\sigma]\sigma}}_{=0\ (\text{traceless})}-2\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\Gamma_\nu\Theta^{(560)\grave{a}}_{\alpha\beta}\underbrace{\Lambda^{(320_a)}_{[\sigma\nu]\sigma}}_{=0\ (\text{traceless})}\\
&\quad+\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}B\Gamma_\nu\Gamma_\mu\Gamma_\lambda\Theta^{(560)\grave{a}}_{\alpha\beta}\underbrace{\Lambda^{(320_a)}_{[\mu\nu]\lambda}}_{-\Lambda^{(320_a)}_{[\nu\mu]\lambda}}\quad[\text{From eq. (E.2)}]\\
&= -\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta}\ B\ \Gamma_\nu\Gamma_\mu\Gamma_\lambda\Theta^{(560)\grave{a}}_{\alpha\beta}\Lambda^{(320_a)}_{[\nu\mu]\lambda}\\
&= -J_{\grave{b}\grave{a}}\implies \text{Conclusion: } J_{aa}=0.\ \textit{This coupling vanishes only for single generation.}
\end{aligned}$$

CASE 2: $\quad X_{\mu\nu\lambda} = \Lambda^{(320_s)}_{(\mu\nu)\lambda}$

$$J_{\grave{a}\grave{b}} = \Theta^{(560)\grave{a}\mathsf{T}}_{\alpha\beta}B\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Theta^{(560)\grave{b}}_{\alpha\beta}\Lambda^{(320_s)}_{(\mu\nu)\lambda}\quad[\text{From eq. (E.1)}]$$

$$= 2\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta} B\Gamma_\mu \Theta^{(560)\grave{a}}_{\alpha\beta} \underbrace{\Lambda^{(320_s)}_{(\mu\sigma)\sigma}}_{=0\ \text{(traceless)}} - 2\Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta} B\Gamma_\nu \Theta^{(560)\grave{a}}_{\alpha\beta} \underbrace{\Lambda^{(320_s)}_{(\sigma\nu)\sigma}}_{=0\ \text{(traceless)}}$$

$$+ \Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta} B\Gamma_\nu\Gamma_\mu\Gamma_\lambda \Theta^{(560)\grave{a}}_{\alpha\beta} \Lambda^{(320_s)}_{(\mu\nu)\lambda} \quad \text{[From eq. (E.2)]}$$

$$= \Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta} B \underbrace{\Gamma_\nu\Gamma_\mu}_{=\delta_{\nu\mu}+\Sigma_{\nu\mu}\ \ \text{[see **]}} \Gamma_\lambda \Theta^{(560)\grave{a}}_{\alpha\beta} \Lambda^{(320_s)}_{(\mu\nu)\lambda}$$

$$= \Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta} B\Gamma_\lambda \Theta^{(560)\grave{a}}_{\alpha\beta} \underbrace{\Lambda^{(320_s)}_{(\sigma\sigma)\lambda}}_{=0\ \text{(traceless)}} + \Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta} B\Sigma_{\nu\mu}\Gamma_\lambda \Theta^{(560)\grave{a}}_{\alpha\beta} \Lambda^{(320_s)}_{(\mu\nu)\lambda}$$

$$= \Theta^{(560)\grave{b}\mathsf{T}}_{\alpha\beta} B\Sigma_{\nu\mu}\Gamma_\lambda \Theta^{(560)\grave{a}}_{\alpha\beta} \Lambda^{(320_s)}_{(\mu\nu)\lambda}$$

$= 0$ [since Σ matrix is antisymmetric in its indices and 320 tensor is symmetric in its first two indices]

Conclusion: $J_{\grave{a}\grave{b}} = 0, \forall \grave{a}, \grave{b}$. *This coupling vanishes for both single & multiple generations*

$$* \quad \Gamma_\lambda\Gamma_\nu\Gamma_\mu = \left(-\Gamma_\nu\Gamma_\lambda + 2\delta_{\nu\lambda}\right)\Gamma_\mu = 2\delta_{\nu\lambda}\Gamma_\mu - \Gamma_\nu\left(2\delta_{\mu\lambda} - \Gamma_\mu\Gamma_\lambda\right) = 2\delta_{\nu\lambda}\Gamma_\mu - 2\delta_{\mu\lambda}\Gamma_\nu + \Gamma_\nu\Gamma_\mu\Gamma_\lambda$$

$$** \quad \Gamma_\nu\Gamma_\mu = \underbrace{\frac{1}{2}\left(\Gamma_\nu\Gamma_\mu + \Gamma_\mu\Gamma_\nu\right)}_{=\delta_{\nu\mu}} + \underbrace{\frac{1}{2}\left(\Gamma_\nu\Gamma_\mu - \Gamma_\mu\Gamma_\nu\right)}_{=\Sigma_{\nu\mu}} = \delta_{\nu\mu} + \underbrace{\Sigma_{\nu\mu}}_{=-\Sigma_{\mu\nu}}$$

E.2 Couplings that involve a single gamma

Consider the coupling defined through

$$I_{\grave{a}\grave{b}} = \Theta^{(560)\grave{a}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{b}}_{\nu\sigma} X_{\mu\nu\lambda}, \tag{E.3}$$

where $X_{\mu\nu\lambda}$ is a generic three-index tensor. Therefore

$$\begin{aligned} I_{\grave{a}\grave{b}} &= \left(\Theta^{(560)\grave{a}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda\ \Theta^{(560)\grave{b}}_{\nu\sigma}\right)^{\mathsf{T}} X_{\mu\nu\lambda} \\ &= \Theta^{(560)\grave{b}\mathsf{T}}_{\nu\sigma} \Gamma^{\mathsf{T}}_\lambda \underbrace{B^{\mathsf{T}}}_{=-B} \left(\Theta^{(560)\grave{a}\mathsf{T}}_{\mu\sigma}\right)^{\mathsf{T}} X_{\mu\nu\lambda} \\ &= -\Theta^{(560)\grave{b}\mathsf{T}}_{\nu\sigma} \Gamma^{\mathsf{T}}_\lambda B \Theta^{(560)\grave{a}}_{\mu\sigma} X_{\mu\nu\lambda} \\ &= -\Theta^{(560)\grave{b}\mathsf{T}}_{\nu\sigma} B \underbrace{\left(B^{-1}\Gamma^{\mathsf{T}}_\lambda B\right)}_{=-\Gamma_\lambda} \Theta^{(560)\grave{a}}_{\mu\sigma} X_{\mu\nu\lambda} \\ &= +\Theta^{(560)\grave{b}\mathsf{T}}_{\nu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{a}}_{\mu\sigma} X_{\mu\nu\lambda} \\ &= \Theta^{(560)\grave{b}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda\ \Theta^{(560)\grave{a}}_{\nu\sigma} X_{\nu\mu\lambda} \quad \mu,\nu \text{ (dummy indices) interchange} \end{aligned} \tag{E.4}$$

CASE 1(a): $\quad X_{\mu\nu\lambda} = \Lambda^{(320_s)}_{(\mu\nu)\lambda}$

$$\begin{aligned} I_{\grave{a}\grave{b}} &= \Theta^{(560)\grave{a}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{b}}_{\nu\sigma} \Lambda^{(320_s)}_{(\mu\nu)\lambda} && \text{[From eq. (E.3)]} \\ &= \Theta^{(560)\grave{b}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{a}}_{\nu\sigma} \Lambda^{(320_s)}_{(\nu\mu)\lambda} && \text{[From eq. (E.4)]} \end{aligned}$$

$$= \Theta^{(560)\grave{b}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{a}}_{\nu\sigma} \Lambda^{(320_s)}_{(\mu\nu)\lambda}$$

$$= +\, I_{\grave{b}\grave{a}} \implies \text{Conclusion: } I_{\grave{a}\grave{a}} \neq 0.$$ *This coupling is non-vanishing for both single & multiple generations*

CASE 1(b): $X_{\mu\nu\lambda} = \Lambda^{(320_s)}_{(\mu\lambda)\nu}$

$$I_{\grave{a}\grave{b}} = \Theta^{(560)\grave{a}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{b}}_{\nu\sigma} \Lambda^{(320_s)}_{(\mu\lambda)\nu} \qquad \text{[From eq. (E.3)]}$$
$$= \Theta^{(560)\grave{b}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{a}}_{\nu\sigma} \Lambda^{(320_s)}_{(\nu\lambda)\mu} \qquad \text{[From eq. (E.4)]}$$

No symmetry in the exchange of 560's

Conclusion: *This coupling is non-vanishing for both single & multiple generations*

CASE 2(a): $X_{\mu\nu\lambda} = \Lambda^{(320_a)}_{[\mu\nu]\lambda}$

$$I_{\grave{a}\grave{b}} = \Theta^{(560)\grave{a}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{b}}_{\nu\sigma} \Lambda^{(320_a)}_{[\mu\nu]\lambda} \qquad \text{[From eq. (E.3)]}$$
$$= \Theta^{(560)\grave{b}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{a}}_{\nu\sigma} \Lambda^{(320_a)}_{[\nu\mu]\lambda} \qquad \text{[From eq. (E.4)]}$$
$$= -\Theta^{(560)\grave{b}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{a}}_{\nu\sigma} \Lambda^{(320_a)}_{[\mu\nu]\lambda}$$
$$= -I_{\grave{b}\grave{a}} \implies \text{Conclusion: } I_{aa} = 0.$$ *This coupling vanishes only for single generation*

CASE 2(b): $X_{\mu\nu\lambda} = \Lambda^{(320_a)}_{[\mu\lambda]\nu}$

$$I_{\grave{a}\grave{b}} = \Theta^{(560)\grave{a}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{b}}_{\nu\sigma} \Lambda^{(320_a)}_{[\mu\lambda]\nu} \qquad \text{[From eq. (E.3)]}$$
$$= \Theta^{(560)\grave{b}\mathsf{T}}_{\mu\sigma} B\Gamma_\lambda \Theta^{(560)\grave{a}}_{\nu\sigma} \Lambda^{(320_a)}_{[\nu\lambda]\mu} \qquad \text{[From eq. (E.4)]}$$

No symmetry in the exchange of 560's

Conclusion: *This coupling is non-vanishing for both single & multiple generations*

F Construction of normalized 70-, 45- and 5-plets with diagonal kinetic energy terms contained in $\Lambda^{320_s}_{(\mu\nu)\lambda}$ of SO(10)

F.1 STEP 1: identifying three 70's, two 45's and eight 5's in $\Lambda^{320_s}_{(\mu\nu)\lambda}$

$$\Lambda^{320_s}_{(\mu\nu)\lambda} = t_{(\mu\nu)\lambda} + \frac{1}{54}(\delta_{\nu\lambda}\delta_{\mu\beta} + \delta_{\mu\lambda}\delta_{\nu\beta} - 2\delta_{\mu\nu}\delta_{\lambda\beta})\tau_\beta \tag{F.1}$$

where

$$t_{(\mu\nu)\lambda} \equiv \frac{1}{6}\left[2\left(T_{\mu\nu\lambda} + T_{\nu\mu\lambda}\right) - \left(T_{\lambda\mu\nu} + T_{\lambda\nu\mu} + T_{\mu\lambda\nu} + T_{\nu\lambda\mu}\right)\right] \tag{F.2a}$$
$$\tau_\beta \equiv -T_{\beta\alpha\alpha} + 2T_{\alpha\alpha\beta} - T_{\alpha\beta\alpha} \tag{F.2b}$$

JHEP07(2025)222

We define the eight distinct $\mathsf{5}-$plets of $\mathsf{SU(5)}$ as follows:

$$
\begin{aligned}
T_{c_i c_n \bar{c}_n} + T_{c_n c_i \bar{c}_n} &\equiv H^{(5_1)i}, \\
T_{\bar{c}_n c_i c_n} + T_{\bar{c}_n c_n c_i} &\equiv H^{(5_2)i}, \\
T_{c_i \bar{c}_n c_n} + T_{c_n \bar{c}_n c_i} &\equiv H^{(5_3)i}, \\
\\
T_{\bar{c}_n c_n c_i} - T_{\bar{c}_n c_i c_n} &\equiv H^{(5'_2)i}, \\
T_{c_n \bar{c}_n c_i} - T_{c_i \bar{c}_n c_n} &\equiv H^{(5'_3)i}, \\
\\
T_{c_i c_n \bar{c}_n} + T_{c_i \bar{c}_n c_n} &\equiv H^{(5''_1)i}, \\
T_{c_n c_i \bar{c}_n} + T_{\bar{c}_n c_i c_n} &\equiv H^{(5''_2)i}, \\
T_{c_n \bar{c}_n c_i} + T_{\bar{c}_n c_n c_i} &\equiv H^{(5''_3)i}.
\end{aligned}
\tag{F.3}
$$

while the three $\mathsf{70}-$plets of $\mathsf{SU(5)}$ are defined through

$$
\begin{aligned}
T_{c_i c_j \bar{c}_k} + T_{c_j c_i \bar{c}_k} &= H_k^{(70_1)ij} + \frac{1}{6}\left[\delta^i_k H^{(5_1)j} + \delta^j_k H^{(5_1)i}\right], \\
T_{\bar{c}_k c_i c_j} + T_{\bar{c}_k c_j c_i} &= H_k^{(70_2)ij} + \frac{1}{6}\left[\delta^i_k H^{(5_2)j} + \delta^j_k H^{(5_2)i}\right], \\
T_{c_i \bar{c}_k c_j} + T_{c_j \bar{c}_k c_i} &= H_k^{(70_3)ij} + \frac{1}{6}\left[\delta^i_k H^{(5_3)j} + \delta^j_k H^{(5_3)i}\right].
\end{aligned}
\tag{F.4}
$$

and finally the two distinct $\mathsf{45}-$plets of $\mathsf{SU(5)}$ are defined by

$$
\begin{aligned}
T_{\bar{c}_i c_j c_k} - T_{\bar{c}_i c_k c_j} &= H_i^{(45_2)jk} + \frac{1}{4}\left[\delta^j_i H^{(5'_2)k} + \delta^k_i H^{(5'_2)j}\right], \\
T_{c_j \bar{c}_i c_k} - T_{c_k \bar{c}_i c_j} &= H_i^{(45_3)jk} + \frac{1}{4}\left[\delta^j_i H^{(5'_3)k} + \delta^k_i H^{(5'_3)j}\right].
\end{aligned}
\tag{F.5}
$$

One may now very easily define $\overline{\mathsf{5}}-$plets, $\overline{\mathsf{45}}-$plets, $\overline{\mathsf{70}}-$plets of $\mathsf{SU(5)}$. The reducible $\mathsf{SU(5)}$ tensors given by eq. (F.2a)

$$
\begin{aligned}
t_{(c_i c_j)\bar{c}_k} = \frac{1}{6}\Bigg\{ & 2\left[H_k^{(70_1)ij} + \frac{1}{6}\left(\delta^i_k H^{(5_1)j} + \delta^j_k H^{(5_1)i}\right)\right] - \left[H_k^{(70_2)ij} + \frac{1}{6}\left(\delta^i_k H^{(5_2)j} + \delta^j_k H^{(5_2)i}\right)\right] \\
& - \left[H_k^{(70_3)ij} + \frac{1}{6}\left(\delta^i_k H^{(5_3)j} + \delta^j_k H^{(5_3)i}\right)\right]\Bigg\}, \\
t_{(\bar{c}_i c_j)c_k} = \frac{1}{6}\Bigg\{ & \frac{3}{2}\left[H_i^{(45_2)jk} + \frac{1}{4}\left(\delta^j_i H^{(5'_2)k} - \delta^k_i H^{(5'_2)j}\right)\right] + \frac{3}{2}\left[H_i^{(45_3)jk} + \frac{1}{4}\left(\delta^j_i H^{(5'_3)k} - \delta^k_i H^{(5'_3)j}\right)\right] \\
& - \left[H_i^{(70_1)jk} + \frac{1}{6}\left(\delta^j_i H^{(5_1)k} + \delta^k_i H^{(5_1)j}\right)\right] + \frac{1}{2}\left[H_i^{(70_2)jk} + \frac{1}{6}\left(\delta^j_i H^{(5_2)k} + \delta^k_i H^{(5_2)j}\right)\right] \\
& + \frac{1}{2}\left[H_i^{(70_3)jk} + \frac{1}{6}\left(\delta^j_i H^{(5_3)k} + \delta^k_i H^{(5_3)j}\right)\right]\Bigg\}.
\end{aligned}
\tag{F.6}
$$

Note that the tensors $t_{(\bar{c}_i \bar{c}_j)c_k}$ and $t_{(c_i \bar{c}_j)\bar{c}_k}$ would have same expressions except the unbarred $\mathsf{SU(5)}$ fields would be replaced by barred ones. Finally, we do not compute $t_{(c_i c_j)c_k}$ and $t_{(\bar{c}_i \bar{c}_j)\bar{c}_k}$ as they contain $\mathsf{40}-$plet and $\overline{\mathsf{40}}-$plet of $\mathsf{SU(5)}$ which does not participate in the doublet-triplet splitting. This is because $\mathsf{40} + \overline{\mathsf{40}}$ multiplets do not share the same $\mathsf{U(1)_Y}$ hypercharge of ± 3 as the $\mathsf{5} + \overline{\mathsf{5}}$ multiplets of $\mathsf{SU(5)}$.

F.2 STEP 2: construction of the kinetic energy of $\Lambda^{320_s}_{(\mu\nu)\lambda}$ multiplet

$$\begin{aligned}
\mathcal{L}^{(320_s)}_{\rm KE} =& -\partial_A\Lambda^{320_s}_{(\mu\nu)\lambda}\partial^A\Lambda^{320_s\dagger}_{(\mu\nu)\lambda} \\
=& -\left[\partial_A t_{(\mu\nu)\lambda}\partial^A t^\dagger_{(\mu\nu)\lambda} + \frac{1}{54^2}\left(\delta_{\nu\lambda}\partial_A\tau_\mu + \delta_{\mu\lambda}\partial_A\tau_\nu - 2\delta_{\mu\nu}\partial_A\tau_\lambda\right)\left(\delta_{\nu\lambda}\partial^A\tau^\dagger_\mu + \delta_{\mu\lambda}\partial^A\tau^\dagger_\nu - 2\delta_{\mu\nu}\partial^A\tau^\dagger_\lambda\right)\right. \\
&\left. + \frac{1}{54}\partial_A t_{(\mu\nu)\lambda}\left(\delta_{\nu\lambda}\partial^A\tau^\dagger_\mu + \delta_{\mu\lambda}\partial^A\tau^\dagger_\nu - 2\delta_{\mu\nu}\partial^A\tau^\dagger_\lambda\right) + \frac{1}{54}\left(\delta_{\nu\lambda}\partial_A\tau_\mu + \delta_{\mu\lambda}\partial_A\tau_\nu - 2\delta_{\mu\nu}\partial_A\tau_\lambda\right)\partial^A t^\dagger_{(\mu\nu)\lambda}\right] \\
=& -\frac{1}{8}\left[\partial_A t_{(c_ic_j)c_k}\partial^A t^\dagger_{(c_ic_j)c_k} + \partial_A t_{(\bar{c}_i\bar{c}_j)\bar{c}_k}\partial^A t^\dagger_{(\bar{c}_i\bar{c}_j)\bar{c}_k} + \partial_A t_{(c_ic_j)\bar{c}_k}\partial^A t^\dagger_{(c_ic_j)\bar{c}_k} + \partial_A t_{(\bar{c}_i\bar{c}_j)c_k}\partial^A t^\dagger_{(\bar{c}_i\bar{c}_j)c_k}\right. \\
&\left. + 2\partial_A t_{(\bar{c}_ic_j)c_k}\partial^A t^\dagger_{(\bar{c}_ic_j)c_k} + 2\partial_A t_{(\bar{c}_ic_j)\bar{c}_k}\partial^A t^\dagger_{(\bar{c}_ic_j)\bar{c}_k}\right] + \frac{1}{54}\partial_A\tau_\mu\partial^A\tau^\dagger_\mu \\
=& \mathcal{L}^{(5_{320})}_{\rm KE} + \mathcal{L}^{(45_{320})}_{\rm KE} + \mathcal{L}^{(70_{320})}_{\rm KE} + \cdots,
\end{aligned} \tag{F.7}$$

where $\cdots$ represent $\mathcal{L}^{(\overline{5}_{320})}_{\rm KE}$, $\mathcal{L}^{(\overline{45}_{320})}_{\rm KE}$, $\mathcal{L}^{(\overline{70}_{320})}_{\rm KE}$, $\mathcal{L}^{(40_{320})}_{\rm KE}$ and $\mathcal{L}^{(\overline{40}_{320})}_{\rm KE}$. As mentioned earlier, the doublets contained in $40+\overline{40}$ do not participate in doublet triplet splitting as their hypercharge are different from $5+\overline{5}$. Thus,

$$\begin{aligned}
\mathcal{L}^{(70_{320})}_{\rm KE} =& -\frac{1}{192}\left[4\partial_A H^{(70_1)ij}_k\partial^A H^{(70_1)ij\dagger}_k + \partial_A H^{(70_2)ij}_k\partial^A H^{(70_2)ij\dagger}_k + \partial_A H^{(70_3)ij}_k\partial^A H^{(70_3)ij\dagger}_k\right. \\
& - 2\partial_A H^{(70_1)ij}_k\partial^A H^{(70_2)ij\dagger}_k - 2\partial_A H^{(70_2)ij}_k\partial^A H^{(70_1)ij\dagger}_k - 2\partial_A H^{(70_1)ij}_k\partial^A H^{(70_3)ij\dagger}_k \\
&\left. - 2\partial_A H^{(70_3)ij}_k\partial^A H^{(70_1)ij\dagger} + \partial_A H^{(70_2)ij}_k\partial^A H^{(70_3)ij\dagger}_k + \partial_A H^{(70_3)ij}_k H^{(70_2)ij\dagger}_k\right]
\end{aligned} \tag{F.8a}$$

$$\begin{aligned}
\mathcal{L}^{(45_{320})}_{\rm KE} =& -\frac{1}{64}\left[\partial_A H^{(45_2)ij}_k\partial^A H^{(45_2)ij\dagger}_k + \partial_A H^{(45_3)ij}_k\partial^A H^{(45_3)ij\dagger}_k\right. \\
&\left. + \partial_A H^{(45_2)ij}_k\partial^A H^{(45_3)ij\dagger}_k + \partial_A H^{(45_3)ij}_k\partial^A H^{(45_2)ij\dagger}_k\right]
\end{aligned} \tag{F.8b}$$

$$\begin{aligned}
\mathcal{L}^{(5_{320})}_{\rm KE} =& -\frac{1}{3456}\left[-8\partial_A H^{(5''_1)i}\partial^A H^{(5''_1)i\dagger} - 8\partial_A H^{(5''_2)i}\partial^A H^{(5''_2)i\dagger} - 32\partial_A H^{(5''_3)i}\partial^A H^{(5''_3)i\dagger}\right. \\
& + 24\partial_A H^{(5_1)i}\partial^A H^{(5_1)i\dagger} + 6\partial_A H^{(5_2)i}\partial^A H^{(5_2)i\dagger} + 6\partial_A H^{(5_3)i}\partial^A H^{(5_3)i\dagger} \\
& + 27\partial_A H^{(5'_2)i}\partial^A H^{(5'_2)i\dagger} + 27\partial_A H^{(5'_3)i}\partial^A H^{(5'_3)i\dagger} \\
& - 8\partial_A H^{(5''_1)i}\partial^A H^{(5''_2)i\dagger} - 8\partial_A H^{(5''_2)i}\partial^A H^{(5''_1)i\dagger} + 16\partial_A H^{(5''_1)i}\partial^A H^{(5''_3)i\dagger} \\
& + 16\partial_A H^{(5''_3)i}\partial^A H^{(5''_1)i\dagger} + 16\partial_A H^{(5''_2)i}\partial^A H^{(5''_3)i\dagger} + 16\partial_A H^{(5''_3)i}\partial^A H^{(5''_2)i\dagger} \\
& - 12\partial_A H^{(5_1)i}\partial^A H^{(5_2)i\dagger} - 12\partial_A H^{(5_2)i}\partial^A H^{(5_1)i\dagger} - 12\partial_A H^{(5_1)i}\partial^A H^{(5_3)i\dagger} \\
& - 12\partial_A H^{(5_3)i}\partial^A H^{(5_1)i\dagger} + 6\partial_A H^{(5_2)i}\partial^A H^{(5_3)i\dagger} + 6\partial_A H^{(5_3)i}\partial^A H^{(5_2)i\dagger} \\
&\left. + 27\partial_A H^{(5'_2)i}\partial^A H^{(5'_3)i\dagger} + 27\partial_A H^{(5'_3)i}\partial^A H^{(5'_2)i\dagger}\right]
\end{aligned} \tag{F.8c}$$

F.3 General case of normalization of the kinetic energy after diagonalization

Consider a set of fields ϕ_i $(i = 1, \ldots, n)$ which have mixed kinetic terms.

$$\mathcal{L}_{\rm KE} = -\partial_A\phi^*_i K_{ij}\partial^A\phi_j = -\partial_A\Phi^\dagger K\partial^A\Phi,$$

where,

$$\Phi^\dagger = \begin{pmatrix} \phi_1^* & \phi_2^* & \phi_3^* & \cdots & \phi_n^* \end{pmatrix}$$

We make the transformation

$$\Phi = V\xi \quad \text{where} \quad VV^\dagger = V^\dagger V = 1.$$

In this case we have

$$\begin{aligned}\mathcal{L}_{\text{KE}} &= -\partial_A \xi^* V^\dagger K V \partial^A \xi \\ &\equiv -\partial_A \xi^\dagger K_D \partial^A \xi,\end{aligned}$$

where,

$$K_D = V^\dagger K V = \text{diag}(k_1, k_2, \ldots, k_n).$$

Next we make the scale transformation where

$$\xi_i = \begin{cases} \frac{1}{\sqrt{k_i}}\chi_i, & \text{for non-vanishing } k_i \\ \chi_i, & \text{for vanishing } k_i \end{cases}$$
$$\text{and} \quad k_i \xi_i^* \xi_i = \chi_i^* \chi_i.$$

Thus,

$$\mathcal{L}_{\text{KE}} = -\partial_A \chi_i^* \partial^A \chi_i,$$

where,

$$\phi_i = V_{ij}\xi_j = \frac{1}{\sqrt{k_j}} V_{ij}\chi_j.$$

Here χ_i are the fields which have diagonal and normalized kinetic energy terms.

F.4 STEP 3: diagonalization and normalization of kinetic energy of 70, 45 and 5 multiplets

F.4.1 $\chi_k^{(70_{320})ij}$: normalized 70-plet with diagonal kinetic energy terms

Write eq. (F.8a) as

$$\mathcal{L}_{kin}^{(70_{320})} = -\partial_A \Phi^{(70)\dagger} K^{(70)} \partial^A \Phi^{(70)},$$

where

$$\Phi^{(70)\dagger} = \begin{pmatrix} H_k^{(70_1)ij\dagger} & H_k^{(70_2)ij\dagger} & H_k^{(70_3)ij\dagger} \end{pmatrix}$$

and

$$K^{(70)} = \frac{1}{192}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}.$$

The diagonal matrix is

$$K_D^{(70)} = \begin{pmatrix} \frac{1}{32} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

while the unitary diagonalizing matrix is

$$V^{(70)} = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{5}} & \sqrt{\frac{2}{15}} \\ \frac{1}{\sqrt{6}} & 0 & \sqrt{\frac{5}{6}} \\ \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{30}} \end{pmatrix}.$$

The scaled unitary transformed fields are

$$\begin{aligned} H_k^{(70_1)ij} &= 4\sqrt{2}\, V_{11}^{(70)}\, \chi_k^{(70_{320})ij}, & V_{11}^{(70)} &= -\sqrt{\frac{2}{3}} \\ H_k^{(70_2)ij} &= 4\sqrt{2}\, V_{21}^{(70)}\, \chi_k^{(70_{320})ij}, & V_{21}^{(70)} &= \frac{1}{\sqrt{6}} \\ H_k^{(70_3)ij} &= 4\sqrt{2}\, V_{31}^{(70)}\, \chi_k^{(70_{320})ij}, & V_{31}^{(70)} &= \frac{1}{\sqrt{6}} \end{aligned} \tag{F.9}$$

F.4.2 $\chi_k^{(45_{320})ij}$: normalized 45-plet with diagonal kinetic energy terms

Similarly, eq. (F.8b) can be written as

$$\mathcal{L}_{kin}^{(45_{320})} = -\partial_A \Phi^{(45)\dagger} K^{(45)} \partial^A \Phi^{(45)},$$

where,

$$\Phi^{(45)\dagger} = \begin{pmatrix} H_k^{(45_3)ij\dagger} & H_k^{(45_3)ij\dagger} \end{pmatrix}$$

and

$$K^{(45)} = \frac{1}{64}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

The diagonal matrix is

$$K_D^{(45)} = \begin{pmatrix} \frac{1}{32} & 0 \\ 0 & 0 \end{pmatrix}$$

while the unitary diagonalizing matrix is

$$V^{(45)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

The scaled unitary transformed fields are

$$\begin{aligned} H_k^{(45_2)ij} &= 4\sqrt{2}\, V_{11}^{(45)}\, \chi_k^{(45_{320})ij}, \qquad V_{11}^{(45)} = \frac{1}{\sqrt{2}} \\ H_k^{(45_3)ij} &= 4\sqrt{2}\, V_{21}^{(45)}\, \chi_k^{(45_{320})ij}, \qquad V_{21}^{(45)} = \frac{1}{\sqrt{2}} \end{aligned} \tag{F.10}$$

F.4.3 $\chi^{(5_{320})i}$: normalized 5-plet with diagonal kinetic energy terms

Before we diagonalize the kinetic energy matrix for the 5−plet, let us define

$$\begin{aligned} H_i^{(5_1'')} &= T_{c_i c_n \bar{c}_n} + T_{c_i \bar{c}_n c_n} \equiv h_1^i + h_2^i, \qquad H_i^{(5_2'')} = T_{c_n c_i \bar{c}_n} + T_{\bar{c}_n c_i c_n} \equiv h_3^i + h_4^i, \\ H_i^{(5_3'')} &= T_{c_n \bar{c}_n c_i} + T_{\bar{c}_n c_n c_i} \equiv h_5^i + h_6^i, \end{aligned}$$

then

$$\begin{aligned} H^{(5_1)i} &= T_{c_i c_n \bar{c}_n} + T_{c_n c_i \bar{c}_n} = h_1^i + h_3^i, \qquad H^{(5_2)i} = T_{\bar{c}_n c_i c_n} + T_{\bar{c}_n c_n c_i} = h_4^i + h_6^i, \\ H^{(5_3)i} &= T_{c_i \bar{c}_n c_n} + T_{c_n \bar{c}_n c_i} = h_2^i + h_5^i, \\ H^{(5_2')k} &= T_{\bar{c}_n c_n c_k} - T_{\bar{c}_n c_k c_n} = h_6^i - h_4^i, \qquad H^{(5_3')i} = T_{c_n \bar{c}_n c_k} - T_{c_k \bar{c}_n c_n} = h_5^i - h_2^i \end{aligned}$$

In terms of h^i fields, the kinetic energy of the 5−plets from eq. (F.8c) can be rewritten as

$$\begin{aligned} \mathcal{L}_{kin}^{(5_{320})} = -\frac{1}{3456}\Big[& 16\partial_A h_1^i \partial^A h_1^{i\dagger} - 20\partial_A h_2^i \partial^A h_1^{i\dagger} + 16\partial_A h_3^i \partial^A h_1^{i\dagger} - 20\partial_A h_4^i \partial^A h_1^{i\dagger} + 4\partial_A h_5^i \partial^A h_1^{i\dagger} \\ & + 4\partial_A h_6^i \partial^A h_1^{i\dagger} - 20\partial_A h_1^i \partial^A h_2^{i\dagger} + 25\partial_A h_2^i \partial^A h_2^{i\dagger} - 20\partial_A h_3^i \partial^A h_2^{i\dagger} + 25\partial_A h_4^i \partial^A h_2^{i\dagger} \\ & - 5\partial_A h_5^i \partial^A h_2^{i\dagger} - 5\partial_A h_6^i \partial^A h_2^{i\dagger} + 16\partial_A h_1^i \partial^A h_3^{i\dagger} - 20\partial_A h_2^i \partial^A h_3^{i\dagger} + 16\partial_A h_3^i \partial^A h_3^{i\dagger} \\ & - 20\partial_A h_4^i \partial^A h_3^{i\dagger} + 4\partial_A h_5^i \partial^A h_3^{i\dagger} + 4\partial_A h_6^i \partial^A h_3^{i\dagger} - 20\partial_A h_1^i \partial^A h_4^{i\dagger} + 25\partial_A h_2^i \partial^A h_4^{i\dagger} \\ & - 20\partial_A h_3^i \partial^A h_4^{i\dagger} + 25\partial_A h_4^i \partial^A h_4^{i\dagger} - 5\partial_A h_5^i \partial^A h_4^{i\dagger} - 5\partial_A h_6^i \partial^A h_4^{i\dagger} + 4\partial_A h_1^i \partial^A h_5^{i\dagger} \\ & - 5\partial_A h_2^i \partial^A h_5^{i\dagger} + 4\partial_A h_3^i \partial^A h_5^{i\dagger} - 5\partial_A h_4^i \partial^A h_5^{i\dagger} + \partial_A h_5^i \partial^A h_5^{i\dagger} + \partial_A h_6^i \partial^A h_5^{i\dagger} \\ & + 4\partial_A h_1^i \partial^A h_6^{i\dagger} - 5\partial_A h_2^i \partial^A h_6^{i\dagger} + 4\partial_A h_3^i \partial^A h_6^{i\dagger} - 5\partial_A h_4^i \partial^A h_6^{i\dagger} + \partial_A h_5^i \partial^A h_6^{i\dagger} + \partial_A h_6^i \partial^A h_6^{i\dagger} \Big] \end{aligned}$$

Now write

$$\mathcal{L}_{kin}^{(5_{320})} = -\partial_A \Phi^{(5)\dagger} K^{(5)} \partial^A \Phi^{(5)},$$

where,

$$\Phi^{(5)\dagger} = \begin{pmatrix} h_1^{i\dagger} & h_2^{i\dagger} & h_3^{i\dagger} & h_4^{i*} & h_5^{i\dagger} & h_6^{i\dagger} \end{pmatrix}$$

and

$$K^{(5)} = \frac{1}{3456}\begin{pmatrix} 16 & -20 & 16 & -20 & 4 & 4 \\ -20 & 25 & -20 & 25 & -5 & -5 \\ 16 & -20 & 16 & -20 & 4 & 4 \\ -20 & 25 & -20 & 25 & -5 & -5 \\ 4 & -5 & 4 & -5 & 1 & 1 \\ 4 & -5 & 4 & -5 & 1 & 1 \end{pmatrix}.$$

The diagonal matrix is

$$K_D^{(5)} = \begin{pmatrix} \frac{7}{288} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

while the unitary diagonalizing matrix is

$$V^{(5)} = \begin{pmatrix} \frac{2}{\sqrt{21}} & -\frac{1}{\sqrt{17}} & -\frac{2}{3}\sqrt{\frac{2}{17}} & \frac{10}{3}\sqrt{\frac{2}{43}} & -\frac{16}{\sqrt{2537}} & \frac{10}{\sqrt{1239}} \\ -\frac{5}{2\sqrt{21}} & 0 & 0 & 0 & 0 & \frac{1}{2}\sqrt{\frac{59}{21}} \\ \frac{2}{\sqrt{21}} & 0 & 0 & 0 & \sqrt{\frac{43}{59}} & \frac{10}{\sqrt{1239}} \\ -\frac{5}{2\sqrt{21}} & 0 & 0 & 3\sqrt{\frac{2}{43}} & \frac{20}{\sqrt{2537}} & -\frac{25}{2\sqrt{1239}} \\ \frac{1}{2\sqrt{21}} & 0 & \frac{1}{3}\sqrt{\frac{17}{2}} & \frac{5}{3\sqrt{86}} & -\frac{4}{\sqrt{2537}} & \frac{5}{2\sqrt{1239}} \\ \frac{1}{2\sqrt{21}} & \frac{4}{\sqrt{17}} & -\frac{1}{3\sqrt{34}} & \frac{5}{3\sqrt{86}} & -\frac{4}{\sqrt{2537}} & \frac{5}{2\sqrt{1239}} \end{pmatrix}$$

Make the scaled unitary transformation:

$$h_1^i = 12\sqrt{\frac{2}{7}}\, V_{11}^{(5)}\, \chi^{(5_{320})i}, \qquad V_{11}^{(5)} = \frac{2}{\sqrt{21}}$$

$$h_2^i = 12\sqrt{\frac{2}{7}}\, V_{21}^{(5)}\, \chi^{(5_{320})i}, \qquad V_{21}^{(5)} = -\frac{5}{2\sqrt{21}}$$

$$h_3^i = 12\sqrt{\frac{2}{7}}\, V_{31}^{(5)}\, \chi^{(5_{320})i}, \qquad V_{31}^{(5)} = \frac{2}{\sqrt{21}}$$

$$h_4^i = 12\sqrt{\frac{2}{7}}\, V_{41}^{(5)}\, \chi^{(5_{320})i}, \qquad V_{41}^{(5)} = -\frac{5}{2\sqrt{21}}$$

$$h_5^i = 12\sqrt{\frac{2}{7}}\, V_{51}^{(5)}\, \chi^{(5_{320})i}, \qquad V_{51}^{(5)} = \frac{1}{2\sqrt{21}}$$
$$h_6^i = 12\sqrt{\frac{2}{7}}\, V_{61}^{(5)}\, \chi^{(5_{320})i}, \qquad V_{61}^{(5)} = \frac{1}{2\sqrt{21}}$$

$$\begin{aligned}
H_i^{(5_1'')} &= 12\sqrt{\frac{2}{7}}\left(V_{11}^{(5)} + V_{21}^{(5)}\right)\chi^{(5_{320})i}, \quad H_i^{(5_2'')} = 12\sqrt{\frac{2}{7}}\left(V_{31}^{(5)} + V_{41}^{(5)}\right)\chi^{(5_{320})i},\\
H_i^{(5_3'')} &= 12\sqrt{\frac{2}{7}}\left(V_{51}^{(5)} + V_{61}^{(5)}\right)\chi^{(5_{320})i},\\
H_i^{(5_1)} &= 12\sqrt{\frac{2}{7}}\left(V_{11}^{(5)} + V_{31}^{(5)}\right)\chi^{(5_{320})i}, \quad H_i^{(5_2)} = 12\sqrt{\frac{2}{7}}\left(V_{41}^{(5)} + V_{61}^{(5)}\right)\chi^{(5_{320})i},\\
H_i^{(5_3)} &= 12\sqrt{\frac{2}{7}}\left(V_{21}^{(5)} + V_{51}^{(5)}\right)\chi^{(5_{320})i},\\
H_i^{(5_2')} &= 12\sqrt{\frac{2}{7}}\left(V_{61}^{(5)} - V_{41}^{(5)}\right)\chi^{(5_{320})i}, \quad H_i^{(5_3')} = 12\sqrt{\frac{2}{7}}\left(V_{51}^{(5)} - V_{21}^{(5)}\right)\chi^{(5_{320})i}.
\end{aligned} \tag{F.11}$$

G Details of doublet couplings

G.1 Computation of $\mathbf{560}\cdot\mathbf{560}\cdot\mathbf{320_s}$ and $\overline{\mathbf{560}}\cdot\overline{\mathbf{560}}\cdot\mathbf{320_s}$ interactions

We will first compute in $560\cdot 560\cdot 320_s$: $\beta M_{\mu\nu\lambda}\Lambda^{(320_s)}_{(\mu\nu)\lambda}$, where

$$M_{\mu\nu\lambda} \equiv \beta\,\langle\Theta^{(560)*}_{\mu\sigma}|B\Gamma_\lambda|\Theta^{(560)}_{\nu\sigma}\rangle$$

Using eq. (F.1), we get

$$\begin{aligned}
&\beta\,\langle\Theta^{(560)*}_{\mu\sigma}|B\Gamma_\lambda|\Theta^{(560)}_{\nu\sigma}\rangle\,\Lambda^{(320_s)}_{(\mu\nu)\lambda}\\
&=\beta\Big\{M_{\mu\nu\lambda}t_{(\mu\nu)\lambda}+\frac{1}{54}\Big(M_{\beta\rho\rho}+M_{\rho\beta\rho}-2M_{\rho\rho\beta}\Big)\Big(-T_{\beta\alpha\alpha}+2T_{\alpha\alpha\beta}-T_{\alpha\beta\alpha}\Big)\Big\}\\
&=\beta\Big\{M_{\mu\nu\lambda}t_{(\mu\nu)\lambda}+\frac{1}{27}M_{\rho\rho c_i}\Big(T_{\bar c_i\alpha\alpha}+T_{\alpha\bar c_i\alpha}-2T_{\alpha\alpha\bar c_i}\Big)+\frac{1}{27}M_{\rho\rho\bar c_i}\Big(T_{c_i\alpha\alpha}+T_{\alpha c_i\alpha}-2T_{\alpha\alpha c_i}\Big)\Big\}\\
&=\beta\Big\{\frac{1}{108}\Big[M_{\rho\rho c_i}\Big(H_i^{(\bar 5_1'')}+H_i^{(\bar 5_2'')}-2H_i^{(\bar 5_3'')}\Big)+M_{\rho\rho\bar c_i}\Big(H^{(5_1'')i}+H^{(5_2'')i}-2H^{(5_3'')i}\Big)\Big]\\
&\qquad+M_{\bar c_i\bar c_j c_k}\,t_{(c_ic_j)\bar c_k}+M_{c_ic_j\bar c_k}\,t_{(\bar c_i\bar c_j)c_k}+2M_{c_i\bar c_j\bar c_k}\,t_{(\bar c_ic_j)c_k}+2M_{\bar c_ic_jc_k}\,t_{(c_i\bar c_j)\bar c_k}+\dots\Big\},
\end{aligned} \tag{G.1}$$

where we have used eq. (F.3). The ellipsis above represent terms containing fields which do not contain $\mathsf{SU(2)_L}$ doublets. Evaluating each of the terms in eq. (G.1) using eqs. (A.11)–

(A.13), (A.19), (F.6), (F.9)–(F.11) we get

$$\frac{1}{108}\Big[M_{\rho\rho c_i}\Big(H_i^{(\overline{5}_1'')}+H_i^{(\overline{5}_2'')}-2H_i^{(\overline{5}_3'')}\Big)+M_{\rho\rho\bar{c}_i}\Big(H^{(5_1'')i}+H^{(5_2'')i}-2H^{(5_3'')i}\Big)\Big]$$
$$=\frac{1}{108}\Big\{\langle\Theta_{\rho\sigma}^{(560)*}|Bb_i|\Theta_{\rho\sigma}^{(560)}\rangle\Big[H_i^{(\overline{5}_1'')}+H_i^{(\overline{5}_2'')}-2H_i^{(\overline{5}_3'')}\Big]$$
$$+\langle\Theta_{\rho\sigma}^{(560)*}|Bb_i^\dagger|\Theta_{\rho\sigma}^{(560)}\rangle\Big[H^{(5_1'')i}+H^{(5_2'')i}-2H^{(5_3'')i}\Big]\Big\}$$
$$=\frac{i}{126\sqrt{2}}\mathbf{H}_i^{(24_{560})k}\mathbf{H}_k^{(45_{560})ij}\chi_j^{(\overline{5}_{320})}-\frac{i}{42\sqrt{6}}\mathbf{H}_{ij}^{(75_{560})kl}\mathbf{H}_k^{(45_{560})ij}\chi_l^{(\overline{5}_{320})}$$
$$-\frac{97i}{63\sqrt{7210}}\mathbf{H}^{(1_{560})}\mathbf{H}_i^{(\overline{5}_{560})}\chi^{(5_{320})i}+\frac{i}{12\sqrt{2163}}\mathbf{H}_j^{(24_{560})i}\mathbf{H}_i^{(\overline{5}_{560})}\chi^{(5_{320})j}$$
$$-\frac{19i}{126\sqrt{145}}\mathbf{H}_k^{(24_{560})i}\mathbf{H}_{ij}^{(\overline{45}_{560})k}\chi^{(5_{320})j}-\frac{i}{84\sqrt{435}}\mathbf{H}_{kl}^{(75_{560})ij}\mathbf{H}_{ij}^{(\overline{45}_{560})k}\chi^{(5_{320})l}$$
$$+\frac{i}{84}\mathbf{H}_k^{(24_{560})i}\mathbf{H}_{ij}^{(\overline{70}_{560})k}\chi^{(5_{320})j}$$

$$M_{\bar{c}_i\bar{c}_jc_k}t_{(c_ic_j)\bar{c}_k}=\langle\Theta_{\bar{c}_i\sigma}^{(560)*}|Bb_k|\Theta_{\bar{c}_j\sigma}^{(560)}\rangle\,t_{(c_ic_j)\bar{c}_k}$$
$$=-i\sqrt{\frac{6}{721}}\mathbf{H}_j^{(24_{560})k}\mathbf{H}_i^{(\overline{5}_{560})}\chi_k^{(70_{320})ij}+2i\sqrt{\frac{2}{145}}\mathbf{H}_l^{(24_{560})j}\mathbf{H}_{jk}^{(\overline{45}_{560})i}\chi_i^{(70_{320})kl}$$
$$+4i\sqrt{\frac{6}{145}}\mathbf{H}_{im}^{(75_{560})jl}\mathbf{H}_{jk}^{(\overline{45}_{560})i}\chi_l^{(70_{320})km}+\frac{i}{3\sqrt{15}}\mathbf{H}^{(1_{560})}\mathbf{H}_{ij}^{(\overline{70}_{560})k}\chi_k^{(70_{320})ij}$$
$$+\frac{i}{3\sqrt{2}}\Big(\mathbf{H}_j^{(24_{560})l}\mathbf{H}_{li}^{(\overline{70}_{560})k}+\mathbf{H}_l^{(24_{560})k}\mathbf{H}_{ij}^{(\overline{70}_{560})l}\Big)\chi_k^{(70_{320})ij}$$
$$+i\sqrt{\frac{2}{3}}\mathbf{H}_{km}^{(75_{560})jl}\mathbf{H}_{ij}^{(\overline{70}_{560})k}\chi_l^{(70_{320})im}+\frac{16i}{7}\sqrt{\frac{2}{3605}}\mathbf{H}^{(1_{560})}\mathbf{H}_i^{(\overline{5}_{560})}\chi^{(5_{320})i}$$
$$+\frac{2i}{7}\sqrt{\frac{3}{721}}\mathbf{H}_j^{(24_{560})i}\mathbf{H}_i^{(\overline{5}_{560})}\chi^{(5_{320})j}+\frac{4i}{7}\sqrt{\frac{5}{29}}\mathbf{H}_k^{(24_{560})i}\mathbf{H}_{ij}^{(\overline{45}_{560})k}\chi^{(5_{320})j}$$
$$-\frac{8i}{7}\sqrt{\frac{3}{145}}\mathbf{H}_{kl}^{(75_{560})ij}\mathbf{H}_{ij}^{(\overline{45}_{560})k}\chi^{(5_{320})l}+\frac{i}{7}\mathbf{H}_i^{(24_{560})k}\mathbf{H}_{ij}^{(\overline{70}_{560})k}\chi^{(5_{320})j}$$

$$M_{c_ic_j\bar{c}_k}t_{(\bar{c}_i\bar{c}_j)c_k}=\langle\Theta_{c_i\sigma}^{(560)*}|B\ b_k^\dagger|\Theta_{c_j\sigma}^{(560)}\rangle\,t_{(\bar{c}_i\bar{c}_j)c_k}$$
$$=\ -\ i\ \mathbf{H}_j^{(24_{560})i}\ \mathbf{H}_l^{(45_{560})jk}\ \chi_{ik}^{(\overline{70}_{320})l}\ +\ \frac{i\sqrt{2}}{7}\ \mathbf{H}_j^{(24_{560})i}\ \mathbf{H}_i^{(45_{560})jk}\ \chi_k^{(\overline{5}_{320})}$$

$$2M_{c_i\bar{c}_j\bar{c}_k}t_{(\bar{c}_ic_j)c_k}=2\,\langle\Theta_{c_i\sigma}^{(560)*}|Bb_k^\dagger|\Theta_{\bar{c}_j\sigma}^{(560)}\rangle\,t_{(\bar{c}_ic_j)c_k}$$
$$=\frac{83i}{7}\sqrt{\frac{2}{3605}}\mathbf{H}^{(1_{560})}\mathbf{H}_i^{(\overline{5}_{560})}\chi^{(5_{320})i}-\frac{305i}{56}\sqrt{\frac{3}{721}}\mathbf{H}_j^{(24_{560})i}\mathbf{H}_i^{(\overline{5}_{560})}\chi^{(5_{320})j}$$
$$+\frac{671i}{84\sqrt{145}}\mathbf{H}_j^{(24_{560})i}\mathbf{H}_{ik}^{(\overline{45}_{560})j}\chi^{(5_{320})k}+\frac{2i}{7\sqrt{435}}\mathbf{H}_{kl}^{(75_{560})ij}\mathbf{H}_{ij}^{(\overline{45}_{560})k}\chi^{(5_{320})l}$$
$$+\frac{2i}{7}\mathbf{H}_j^{(24_{560})i}\mathbf{H}_{ik}^{(\overline{70}_{560})j}\chi^{(5_{320})k}+\frac{31i}{6}\sqrt{\frac{7}{206}}\mathbf{H}_j^{(24_{560})i}\mathbf{H}_k^{(\overline{5}_{560})}\chi_i^{(45_{320})jk}$$

$$
\begin{aligned}
&- \frac{i}{\sqrt{4326}} \mathbf{H}^{(75_{560})ij}_{kl} \mathbf{H}^{(\overline{5}_{560})}_{i} \chi^{(45_{320})kl}_{j} + \frac{49i}{15\sqrt{29}} \mathbf{H}^{(1_{560})} \mathbf{H}^{(\overline{45}_{560})i}_{jk} \chi^{(45_{320})jk}_{i} \\
&+ \frac{i}{\sqrt{870}} \mathbf{H}^{(24_{560})i}_{l} \mathbf{H}^{(\overline{45}_{560})k}_{ij} \chi^{(45_{320})lj}_{k} + \frac{71i}{2\sqrt{870}} \mathbf{H}^{(24_{560})i}_{l} \mathbf{H}^{(\overline{45}_{560})l}_{jk} \chi^{(45_{320})jk}_{i} \\
&+ \frac{i}{2\sqrt{290}} \mathbf{H}^{(75_{560})ij}_{kl} \mathbf{H}^{(\overline{45}_{560})m}_{ij} \chi^{(45_{320})kl}_{m} - \frac{i}{\sqrt{290}} \mathbf{H}^{(75_{560})ij}_{kl} \mathbf{H}^{(\overline{45}_{560})k}_{mi} \chi^{(45_{320})ml}_{j} \\
&+ \frac{7i}{2}\sqrt{\frac{7}{618}} \mathbf{H}^{(24_{560})i}_{j} \mathbf{H}^{(\overline{5}_{560})}_{k} \chi^{(70_{320})jk}_{i} + \frac{i}{3\sqrt{290}} \mathbf{H}^{(24_{560})i}_{j} \mathbf{H}^{(\overline{45}_{560})l}_{ik} \chi^{(70_{320})jk}_{l} \\
&- \frac{i}{\sqrt{870}} \mathbf{H}^{(75_{560})ij}_{kl} \mathbf{H}^{(\overline{45}_{560})l}_{jm} \chi^{(70_{320})km}_{i} - \frac{4i}{3\sqrt{15}} \mathbf{H}^{(1_{560})} \mathbf{H}^{(\overline{70}_{560})i}_{jk} \chi^{(70_{320})jk}_{i} \\
&- \frac{i}{\sqrt{2}} \mathbf{H}^{(24_{560})i}_{j} \mathbf{H}^{(\overline{70}_{560})j}_{kl} \chi^{(70_{320})kl}_{i}
\end{aligned}
$$

$$
\begin{aligned}
2M_{\bar{c}_i c_j c_k}\, t_{(c_i\bar{c}_j)\bar{c}_k} &= 2\,\langle \Theta^{(560)*}_{\bar{c}_i\sigma} | B\ b_k | \Theta^{(560)}_{c_j\sigma}\rangle\, t_{(c_i\bar{c}_j)\bar{c}_k} \\
&= \frac{29i}{84\sqrt{2}}\ \mathbf{H}^{(24_{560})i}_{j}\ \mathbf{H}^{(45_{560})jk}_{i}\ \chi^{(\overline{5}_{320})}_{k} + \frac{5i}{14\sqrt{6}}\ \mathbf{H}^{(75_{560})ij}_{kl}\ \mathbf{H}^{(45_{560})kl}_{i}\ \chi^{(\overline{5}_{320})}_{i} \\
&\quad - \frac{i}{3\sqrt{10}}\ \mathbf{H}^{(1_{560})}\ \mathbf{H}^{(45_{560})ij}_{k}\ \chi^{(\overline{45}_{320})k}_{ij} - \frac{i}{2\sqrt{3}}\ \mathbf{H}^{(24_{560})i}_{j}\ \mathbf{H}^{(45_{560})jk}_{l}\ \chi^{(\overline{45}_{320})l}_{ik} \\
&\quad - \frac{i}{2\sqrt{3}}\ \mathbf{H}^{(24_{560})i}_{j}\ \mathbf{H}^{(45_{560})kl}_{i}\ \chi^{(\overline{45}_{320})j}_{kl} + i\ \mathbf{H}^{(75_{560})ij}_{kl}\ \mathbf{H}^{(45_{560})km}_{j}\ \chi^{(\overline{45}_{320})l}_{im} \\
&\quad + \frac{i}{6}\ \mathbf{H}^{(24_{560})i}_{j}\ \mathbf{H}^{(45_{560})jk}_{l}\ \chi^{(\overline{70}_{320})l}_{ik} + \frac{i}{\sqrt{3}}\ \mathbf{H}^{(75_{560})ij}_{kl}\ \mathbf{H}^{(45_{560})km}_{i}\ \chi^{(\overline{70}_{320})l}_{jm}
\end{aligned}
$$

Using eqs. (B.5), (B.6), (C.2)–(C.4), (C.7), (C.10) in each of the five expressions above, eq. (G.1) gives

$$
\begin{aligned}
\beta\,\langle \Theta^{(560)*}_{\mu\sigma} | B\Gamma_\lambda | \Theta^{(560)}_{\nu\sigma}\rangle\, \Lambda^{(320)}_{(\mu\nu)\lambda} = i\beta\Big\{ &\frac{1}{144\sqrt{2}}\Big(23\sqrt{5}\mathcal{V}_{24} - 16\mathcal{V}_{75}\Big)\, {}^{(45_{560})}\mathbf{D}^{a}\, {}^{(\overline{5}_{320})}\mathbf{D}_{a} \\
&+ \frac{1}{504\sqrt{7210}}\Big(13480\mathcal{V}_{1} + 7761\mathcal{V}_{24}\Big)\, {}^{(\overline{5}_{560})}\mathbf{D}_{a}\, {}^{(5_{320})}\mathbf{D}^{a} \\
&+ \frac{1}{1008\sqrt{29}}\Big(2695\mathcal{V}_{24} + 212\sqrt{5}\mathcal{V}_{75}\Big)\, {}^{(\overline{45}_{560})}\mathbf{D}_{a}\, {}^{(5_{320})}\mathbf{D}^{a} \\
&+ \frac{1}{120\sqrt{174}}\Big(65\mathcal{V}_{24} + 184\sqrt{5}\mathcal{V}_{75}\Big)\, {}^{(\overline{45}_{560})}\mathbf{D}_{a}\, {}^{(70_{320})}\mathbf{D}^{a} \\
&+ \frac{1}{40\sqrt{3}}\Big(-8\sqrt{5}\mathcal{V}_{1} - \sqrt{5}\mathcal{V}_{24} + 20\mathcal{V}_{75}\Big)\, {}^{(\overline{70}_{560})}\mathbf{D}_{a}\, {}^{(70_{320})}\mathbf{D}^{a} \\
&+ \frac{1}{48\sqrt{3}}\Big(-5\sqrt{5}\mathcal{V}_{24} + 16\mathcal{V}_{75}\Big)\, {}^{(45_{560})}\mathbf{D}^{a}\, {}^{(\overline{70}_{320})}\mathbf{D}_{a} \\
&+ \frac{1}{60\sqrt{29}}\Big(196\mathcal{V}_{1} - 129\mathcal{V}_{24} + 2\sqrt{5}\mathcal{V}_{75}\Big)\, {}^{(\overline{45}_{560})}\mathbf{D}_{a}\, {}^{(45_{320})}\mathbf{D}^{a} \\
&+ \frac{1}{24\sqrt{1442}}\Big(217\sqrt{5}\mathcal{V}_{24} - 8\mathcal{V}_{75}\Big)\, {}^{(\overline{5}_{560})}\mathbf{D}_{a}\, {}^{(45_{320})}\mathbf{D}^{a} \\
&+ \frac{1}{48\sqrt{10}}\Big(-16\mathcal{V}_{1} + 33\mathcal{V}_{24}\Big)\, {}^{(45_{560})}\mathbf{D}^{a}\, {}^{(\overline{45}_{320})}\mathbf{D}_{a}
\end{aligned}
$$

$$+\frac{37}{168}\sqrt{\frac{5}{2}}\mathcal{V}_{24}{}^{(\overline{70}_{560})}\mathbf{D}_a{}^{(5_{320})}\mathbf{D}^a$$
$$\left.+\frac{37}{8}\sqrt{\frac{5}{2163}}\mathcal{V}_{24}{}^{(\overline{5}_{560})}\mathbf{D}_a{}^{(70_{320})}\mathbf{D}^a+\cdots\right\} \qquad \text{(G.2)}$$

The results for $\overline{560}\cdot\overline{560}\cdot 320$ can now be easily obtained from $560\cdot 560\cdot 320$ by simply replacing barred with unbarred fields and vice versa:

$$\begin{aligned}
\overline{\beta}\,\langle\overline{\Theta}^{(\overline{560})*}_{\mu\sigma}|B\Gamma_\lambda|\overline{\Theta}^{(\overline{560})}_{\nu\sigma}\rangle\,\Lambda^{(320)}_{(\mu\nu)\lambda} = i\overline{\beta}\Bigg\{ & \frac{1}{144\sqrt{2}}\left(23\sqrt{5}\overline{\mathcal{V}}_{24}-16\overline{\mathcal{V}}_{75}\right){}^{(\overline{45}_{\overline{560}})}\mathbf{D}_a{}^{(5_{320})}\mathbf{D}^a \\
&+\frac{1}{504\sqrt{7210}}\left(13480\overline{\mathcal{V}}_1+7761\overline{\mathcal{V}}_{24}\right){}^{(5_{\overline{560}})}\mathbf{D}^a{}^{(\overline{5}_{320})}\mathbf{D}_a \\
&+\frac{1}{1008\sqrt{29}}\left(2965\overline{\mathcal{V}}_{24}+212\sqrt{5}\overline{\mathcal{V}}_{75}\right){}^{(45_{\overline{560}})}\mathbf{D}^a{}^{(\overline{5}_{320})}\mathbf{D}_a \\
&+\frac{1}{120\sqrt{174}}\left(65\overline{\mathcal{V}}_{24}+184\sqrt{5}\overline{\mathcal{V}}_{75}\right){}^{(45_{\overline{560}})}\mathbf{D}^a{}^{(\overline{70}_{320})}\mathbf{D}_a \\
&+\frac{1}{40\sqrt{3}}\left(-8\sqrt{5}\overline{\mathcal{V}}_1-\sqrt{5}\overline{\mathcal{V}}_{24}+20\overline{\mathcal{V}}_{75}\right){}^{(70_{\overline{560}})}\mathbf{D}^a{}^{(\overline{70}_{320})}\mathbf{D}_a \\
&+\frac{1}{48\sqrt{3}}\left(-5\sqrt{5}\overline{\mathcal{V}}_{24}+16\overline{\mathcal{V}}_{75}\right){}^{(\overline{45}_{\overline{560}})}\mathbf{D}_a{}^{(70_{320})}\mathbf{D}^a \\
&+\frac{1}{60\sqrt{29}}\left(196\overline{\mathcal{V}}_1-129\overline{\mathcal{V}}_{24}+2\sqrt{5}\overline{\mathcal{V}}_{75}\right){}^{(45_{\overline{560}})}\mathbf{D}^a{}^{(\overline{45}_{320})}\mathbf{D}_a \\
&+\frac{1}{24\sqrt{1442}}\left(217\sqrt{5}\overline{\mathcal{V}}_{24}-8\overline{\mathcal{V}}_{75}\right){}^{(5_{\overline{560}})}\mathbf{D}^a{}^{(\overline{45}_{320})}\mathbf{D}_a \\
&+\frac{1}{48\sqrt{10}}\left(-16\overline{\mathcal{V}}_1+33\overline{\mathcal{V}}_{24}\right){}^{(\overline{45}_{\overline{560}})}\mathbf{D}_a{}^{(45_{320})}\mathbf{D}^a \\
&+\frac{37}{168}\sqrt{\frac{5}{2}}\overline{\mathcal{V}}_{24}{}^{(70_{\overline{560}})}\mathbf{D}^a{}^{(\overline{5}_{320})}\mathbf{D}_a \\
&\left.+\frac{37}{8}\sqrt{\frac{5}{2163}}\overline{\mathcal{V}}_{24}{}^{(5_{\overline{560}})}\mathbf{D}^a{}^{(\overline{70}_{320})}\mathbf{D}_a+\cdots\right\}
\end{aligned} \qquad \text{(G.3)}$$

Again the ellipsis above represent terms containing fields which are not $\mathsf{SU}(2)_\mathsf{L}$ doublets.

G.2 Computation of $\mathbf{560} \cdot \mathbf{560} \cdot \mathbf{10}_1$, $\overline{\mathbf{560}} \cdot \overline{\mathbf{560}} \cdot (\mathbf{10}_1 + \mathbf{10}_2)$ and $\mathbf{560} \cdot \overline{\mathbf{560}}$ interactions

Using the Basic Theorem ([33]), we can expand the interaction $\alpha \langle \Theta_{\mu\nu}^{(560)*} | B\Gamma_\rho | \Theta_{\mu\nu}^{(560)} \rangle \, \Omega_\rho^{(10_1)}$ $+ \sum_{r=1}^{2} \overline{\alpha}_r \langle \overline{\Theta}_{\mu\nu}^{(\overline{560})*} | B\Gamma_\rho | \overline{\Theta}_{\mu\nu}^{(\overline{560})} \rangle \, \Omega_\rho^{(10_r)}$ as follows:

$$
\begin{aligned}
&\alpha \langle \Theta_{\mu\nu}^{(560)*} | B\Gamma_\rho | \Theta_{\mu\nu}^{(560)} \rangle \, \Omega_\rho^{(10_1)} + \sum_{r=1}^{2} \overline{\alpha}_r \langle \overline{\Theta}_{\mu\nu}^{(\overline{560})*} | B\Gamma_\rho | \overline{\Theta}_{\mu\nu}^{(\overline{560})} \rangle \, \Omega_\rho^{(10_r)} \\
&= 2\alpha \Bigg[\Big(\langle \Theta_{c_i c_j}^{(560)*} | B b_k | \Theta_{\bar{c}_i \bar{c}_j}^{(560)} \rangle - \langle \Theta_{c_i \bar{c}_j}^{(560)*} | B b_k | \Theta_{c_j \bar{c}_i}^{(560)} \rangle \Big) \Omega_{\bar{c}_k}^{(10_1)} \\
&\quad + \Big(\langle \Theta_{c_i c_j}^{(560)*} | B b_k^\dagger | \Theta_{\bar{c}_i \bar{c}_j}^{(560)} \rangle - \langle \Theta_{c_i \bar{c}_j}^{(560)*} | B b_k^\dagger | \Theta_{c_j \bar{c}_i}^{(560)} \rangle \Big) \Omega_{c_k}^{(10_1)} \Bigg] \\
&\quad + 2 \sum_{r=1}^{2} \overline{\alpha}_r \Bigg[\Big(\langle \overline{\Theta}_{c_i c_j}^{(\overline{560})*} | B b_k | \overline{\Theta}_{\bar{c}_i \bar{c}_j}^{(\overline{560})} \rangle - \langle \overline{\Theta}_{c_i \bar{c}_j}^{(\overline{560})*} | B b_k | \overline{\Theta}_{c_j \bar{c}_i}^{(\overline{560})} \rangle \Big) \Omega_{\bar{c}_k}^{(10_r)} \\
&\quad + \Big(\langle \overline{\Theta}_{c_i c_j}^{(\overline{560})*} | B b_k^\dagger | \overline{\Theta}_{\bar{c}_i \bar{c}_j}^{(\overline{560})} \rangle - \langle \overline{\Theta}_{c_i \bar{c}_j}^{(\overline{560})*} | B b_k^\dagger | \overline{\Theta}_{c_j \bar{c}_i}^{(\overline{560})} \rangle \Big) \Omega_{c_k}^{(10_r)} \Bigg]
\end{aligned}
$$

Using eqs. (A.11)–(A.16), (A.19), (C.1) above, we get

$$
\begin{aligned}
&\alpha \langle \Theta_{\mu\nu}^{(560)*} | B\Gamma_\rho | \Theta_{\mu\nu}^{(560)} \rangle \Omega_\rho^{(10_1)} + \sum_{r=1}^{2} \overline{\alpha}_r \langle \overline{\Theta}_{\mu\nu}^{(\overline{560})*} | B\Gamma_\rho | \overline{\Theta}_{\mu\nu}^{(\overline{560})} \rangle \Omega_\rho^{(10_r)} \\
&= i\alpha \Bigg[\Bigg(\sqrt{2} \mathbf{H}_k^{(45_{560})ij} \mathbf{H}_{ij}^{(75_{560})kl} - \sqrt{\frac{2}{3}} \mathbf{H}_j^{(45_{560})kl} \mathbf{H}_k^{(24_{560})j} \Bigg) \mathbf{H}_l^{(\overline{5}_{10_1})} + \Bigg(-\frac{4}{\sqrt{721}} \mathbf{H}_k^{(\overline{5}_{560})} \mathbf{H}_l^{(24_{560})k} \\
&\quad + 97 \sqrt{\frac{2}{10815}} \mathbf{H}_l^{(\overline{5}_{560})} \mathbf{H}^{(1_{560})} + \frac{1}{\sqrt{145}} \mathbf{H}_{jk}^{(\overline{45}_{560})i} \mathbf{H}_{il}^{(75_{560})jk} + 4 \sqrt{\frac{5}{87}} \mathbf{H}_{kl}^{(\overline{45}_{560})j} \mathbf{H}_j^{(24_{560})k} \\
&\quad - \frac{\sqrt{3}}{2} \mathbf{H}_{kl}^{(\overline{70}_{560})j} \mathbf{H}_j^{(24_{560})k} \Bigg) \mathbf{H}^{(5_{10_1})l} + \ldots \Bigg] \\
&+ i \sum_{r=1}^{2} \overline{\alpha}_r \Bigg[\Bigg(\sqrt{2} \mathbf{H}_{ij}^{(\overline{45}_{\overline{560}})k} \mathbf{H}_{kl}^{(\overline{75}_{\overline{560}})ij} - \sqrt{\frac{2}{3}} \mathbf{H}_{kl}^{(\overline{45}_{\overline{560}})j} \mathbf{H}_j^{(\overline{24}_{\overline{560}})k} \Bigg) \mathbf{H}^{(5_{10_r})l} \\
&\quad + \Bigg(-\frac{4}{\sqrt{721}} \mathbf{H}^{(5_{\overline{560}})k} \mathbf{H}_k^{(\overline{24}_{\overline{560}})l} + 97 \sqrt{\frac{2}{10815}} \mathbf{H}^{(5_{\overline{560}})l} \mathbf{H}^{(\overline{1}_{\overline{560}})} + \frac{1}{\sqrt{145}} \mathbf{H}_i^{(45_{\overline{560}})jk} \mathbf{H}_{jk}^{(\overline{75}_{\overline{560}})il} \\
&\quad + 4 \sqrt{\frac{5}{87}} \mathbf{H}_j^{(45_{\overline{560}})kl} \mathbf{H}_k^{(\overline{24}_{\overline{560}})j} - \frac{\sqrt{3}}{2} \mathbf{H}_j^{(70_{\overline{560}})kl} \mathbf{H}_k^{(\overline{24}_{\overline{560}})j} \Bigg) \mathbf{H}_l^{(\overline{5}_{10_r})} + \ldots \Bigg]
\end{aligned}
$$

Using eqs. (B.5), (B.6), (C.2)–(C.4), (C.7), (C.10) in the expression above, we arrive

$$
\begin{aligned}
&\alpha \langle \Theta_{\mu\nu}^{(560)*} | B\Gamma_\rho | \Theta_{\mu\nu}^{(560)} \rangle \, \Omega_\rho^{(10_1)} + \sum_{r=1}^{2} \overline{\alpha}_r \langle \overline{\Theta}_{\mu\nu}^{(\overline{560})*} | B\Gamma_\rho | \overline{\Theta}_{\mu\nu}^{(\overline{560})} \rangle \, \Omega_\rho^{(10_r)} \\
&= i\alpha \Bigg[\Bigg(-\sqrt{\frac{2}{3}} \mathcal{V}_{75}{}^{(45_{560})} \mathbf{D}^a - \frac{1}{2} \sqrt{\frac{5}{6}} \mathcal{V}_{24}{}^{(45_{560})} \mathbf{D}^a \Bigg)^{(\bar{5}_{10_1})} \mathbf{D}_a + \Bigg(2 \sqrt{\frac{6}{3605}} \mathcal{V}_{24}{}^{(\bar{5}_{560})} \mathbf{D}_a
\end{aligned}
$$

$$
\begin{aligned}
&+97\sqrt{\frac{2}{10815}}\mathcal{V}_1{}^{(\bar{\mathsf{5}}_{\mathsf{560}})}\mathbf{D}_a-\frac{1}{\sqrt{435}}\mathcal{V}_{75}{}^{(\overline{\mathsf{45}}_{\mathsf{560}})}\mathbf{D}_a+\frac{5}{\sqrt{87}}\mathcal{V}_{24}{}^{(\overline{\mathsf{45}}_{\mathsf{560}})}\mathbf{D}_a \\
&-\frac{1}{4}\sqrt{\frac{15}{2}}\mathcal{V}_{24}{}^{(\overline{\mathsf{70}}_{\mathsf{560}})}\mathbf{D}_a\Bigg)^{(\mathsf{5}_{\mathsf{10}_1})}\mathbf{D}^a\Bigg] \\
+i\sum_{r=1}^{2}\overline{\alpha}_r\Bigg[&\Bigg(-\sqrt{\frac{2}{3}}\overline{\mathcal{V}}_{75}{}^{(\overline{\mathsf{45}}_{\overline{\mathsf{560}}})}\mathbf{D}_a-\frac{1}{2}\sqrt{\frac{5}{6}}\overline{\mathcal{V}}_{24}{}^{(\overline{\mathsf{45}}_{\overline{\mathsf{560}}})}\mathbf{D}_a\Bigg)^{(\mathsf{5}_{\mathsf{10}_r})}\mathbf{D}^a+\Bigg(2\sqrt{\frac{6}{3605}}\overline{\mathcal{V}}_{24}{}^{(\mathsf{5}_{\overline{\mathsf{560}}})}\mathbf{D}^a \\
&+97\sqrt{\frac{2}{10815}}\overline{\mathcal{V}}_1{}^{(\mathsf{5}_{\overline{\mathsf{560}}})}\mathbf{D}^a-\frac{1}{\sqrt{435}}\overline{\mathcal{V}}_{75}{}^{(\mathsf{45}_{\overline{\mathsf{560}}})}\mathbf{D}^a+\frac{5}{\sqrt{87}}\overline{\mathcal{V}}_{24}{}^{(\mathsf{45}_{\overline{\mathsf{560}}})}\mathbf{D}^a \\
&-\frac{1}{4}\sqrt{\frac{15}{2}}\overline{\mathcal{V}}_{24}{}^{(\mathsf{70}_{\overline{\mathsf{560}}})}\mathbf{D}^a\Bigg)^{(\bar{\mathsf{5}}_{\mathsf{10}_r})}\mathbf{D}_a\Bigg]
\end{aligned}
\tag{G.4}
$$

Finally, we compute the mass term for the $\mathsf{560}+\overline{\mathsf{560}}$

$$
\begin{aligned}
&M_{560}\ \langle\Theta^{(\mathsf{560})*}_{\mu\nu}|B|\overline{\Theta}^{(\overline{\mathsf{560}})}_{\mu\nu}\rangle \\
&=M_{560}\left[\langle\Theta^{(\mathsf{560})*}_{c_ic_j}|B|\overline{\Theta}^{(\overline{\mathsf{560}})}_{\bar{c}_i\bar{c}_j}\rangle-2\langle\Theta^{(\mathsf{560})*}_{c_i\bar{c}_j}|B|\overline{\Theta}^{(\overline{\mathsf{560}})}_{c_j\bar{c}_i}\rangle+\ldots\right] \\
&=iM_{560}\left[\mathbf{H}_i^{(\bar{\mathsf{5}}_{\mathsf{560}})}\mathbf{H}^{(\mathsf{5}_{\overline{\mathsf{560}}})i}-\frac{1}{2}\mathbf{H}_k^{(\mathsf{45}_{\mathsf{560}})ij}\mathbf{H}_{ij}^{(\overline{\mathsf{45}}_{\overline{\mathsf{560}}})k}+\frac{1}{2}\mathbf{H}_{jk}^{(\overline{\mathsf{45}}_{\mathsf{560}})i}\mathbf{H}_i^{(\mathsf{45}_{\overline{\mathsf{560}}})jk}+\frac{1}{2}\mathbf{H}_{jk}^{(\overline{\mathsf{70}}_{\mathsf{560}})i}\mathbf{H}_i^{(\mathsf{70}_{\overline{\mathsf{560}}})jk}+\ldots\right] \\
&=iM_{560}\left[{}^{(\bar{\mathsf{5}}_{\mathsf{560}})}\mathbf{D}_a{}^{(\mathsf{5}_{\overline{\mathsf{560}}})}\mathbf{D}^a-\frac{1}{2}{}^{(\mathsf{45}_{\mathsf{560}})}\mathbf{D}^a{}^{(\overline{\mathsf{45}}_{\overline{\mathsf{560}}})}\mathbf{D}_a+\frac{1}{2}{}^{(\overline{\mathsf{45}}_{\mathsf{560}})}\mathbf{D}_a{}^{(\mathsf{45}_{\overline{\mathsf{560}}})}\mathbf{D}^a+\frac{1}{2}{}^{(\overline{\mathsf{70}}_{\mathsf{560}})}\mathbf{D}_a{}^{(\mathsf{70}_{\overline{\mathsf{560}}})}\mathbf{D}^a+\ldots\right]
\end{aligned}
\tag{G.5}
$$

Data Availability Statement. This article has no associated data or the data will not be deposited.

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