B – L violating interactions in supersymmetric SO(10) models

Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

B − L violating interactions in supersymmetric SO(10) models

To cite this article: Pran Nath and Raza M Syed 2019 *J. Phys.: Conf. Ser.* **1258** 012014

https://doi.org/10.1088/1742-6596/1258/1/012014

$\mathsf{B}-\mathsf{L}$ violating interactions in supersymmetric SO(10) models

Pran Nath[1] **and Raza M Syed**[1,2]

[1] Department of Physics, Northeastern University, Boston, MA 02115-5000, USA
[2] Department of Physics, American University of Sharjah, P.O. Box 26666, Sharjah, UAE

E-mail: `nath@neu.edu, rsyed@aus.edu`

Abstract. An analysis is given of $\mathsf{B}-\mathsf{L}$ violating interactions in $\mathsf{SO}(10)$ arising from the integration of $\mathsf{10}+\overline{\mathsf{10}}$ of $\mathsf{SU}(5)$ of heavy Higgs. The analysis is done within the well defined missing partner model where the doublet-triplet splitting arises naturally. The Higgs representations of the model consist of two 10-plets, one 120, one 210 plet and a $\mathsf{126}+\overline{\mathsf{126}}$ representations of $\mathsf{SO}(10)$. The $\mathsf{SO}(10)$ symmetry is spontaneously broken by the singlet in the $\mathsf{126}+\overline{\mathsf{126}}$ and 210. In this work we focus on the $\mathsf{B}-\mathsf{L}$ interactions arising from the elimination of $\mathsf{10}+\overline{\mathsf{10}}$ plets of $\mathsf{SU}(5)$. We compute five field and six field $\mathsf{B}-\mathsf{L}$ violating operators. The analysis extends previous analyses where $\mathsf{B}-\mathsf{L}$ violating interactions arising from the elimination of $\mathsf{5}+\bar{\mathsf{5}}$ and $\mathsf{45}+\overline{\mathsf{45}}$ were computed. Specifically we compute dimension 6, 7 and 9 operators. These interactions can generate GUT scale baryogengesis, proton decay, and $n-\bar{n}$ oscillations.

1. Introduction

The study of $\mathsf{B}-\mathsf{L}$ violating interactions has a long history [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. Recently there is renewed interest in $\mathsf{B}-\mathsf{L}$ violating interactions in grand unified models since such interactions allow for GUT scale baryogenesis which is not wiped out by sphaleron interactions [23, 24, 25]. For the case of baryon and lepton number violating interactions which preserve $\mathsf{B}-\mathsf{L}$ the baryogengesis arising from such interactions is erased by sphaleron processes which violate $\mathsf{B}+\mathsf{L}$. However, baryogengesis generated by $\mathsf{B}-\mathsf{L}$ violating interactions will not be wiped out. The simplest R parity conserving $\mathsf{SU}(5)$ grand unified model has only $\mathsf{B}-\mathsf{L}$ preserving interactions and thus cannot lead to GUT scale baryogengesis. However, the group $\mathsf{SO}(10)$ has both $\mathsf{B}-\mathsf{L}$ preserving and $\mathsf{B}-\mathsf{L}$ violating interactions and the latter can lead to baryogengesis. Further, $\mathsf{B}-\mathsf{L}$ violating interactions can generate $n-\bar{n}$ interactions and lead to proton decay with exotic modes which are distinguishable from the baryon and lepton number violation but $\mathsf{B}-\mathsf{L}$ preserving interactions. The missing partner mechanism provides a natural mechanism where one has a doublet-triplet splitting [26, 27, 28, 29, 30]. In [28] a variety of $\mathsf{SO}(10)$ missing partners models were classified. Here we will focus on one specific model which consists of a heavy sector comprised of $\mathsf{126}+\overline{\mathsf{126}}+\mathsf{210}$ of Higgs fields and a light sector consisting of $2\times\mathsf{10}+\mathsf{120}$ of Higgs fields. The heavy sector produces the following pairs of doublets and triplet and anti-triplets: $\mathsf{126}+\overline{\mathsf{126}}+\mathsf{210}$ gives $(2D+3T)+(D+T)$ where the Higgs doublet pair (D) consists of up-type and down-type Higgs doublets, and T consists of Higgs triplet/anti-triplet pair. Thus one has a total of $(3D+4T)$ in the heavy Higgs sector.

[2] Permanent address

In the light sector consisting of $2 \times 10 + 120$ of Higgs fields, one has $(2D + 2T) + (2D + 2T)$ set of doublet and triplet/anti-triplet pairs so that one has a total of $(4D + 4T)$ doublet and triplet/anti-triplet pairs of light fields. Now the light and heavy sectors mix via the following couplings in the superpotential $W^{(i)}_{DT} = \mathsf{10}_i \cdot \mathsf{126} \cdot \mathsf{210} + \mathsf{10}_i \cdot \overline{\mathsf{126}} \cdot \mathsf{210} + \mathsf{120} \cdot \mathsf{126} \cdot \mathsf{210} + \mathsf{120} \cdot \overline{\mathsf{126}} \cdot \mathsf{210}$. The mixing of the light and heavy sector then make all the light triplets/anti-triplets in the light sector heavy, while only three of the Higgs doublet pairs in the light sector become heavy leaving one Higgs doublet pair light. It is this residual light Higgs pair that enters in the breaking of the electroweak symmetry. We note here that models of the type we consider here involve large Higgs representations which have a long history. They were discussed in early works in [31] and symmetry breaking in these models was investigated in [32] where a cubic equation for spontaneous symmetry breaking was obtained. Further, extensions and applications of these models were made in a number of works [33, 34].

The $|\Delta(\mathsf{B}-\mathsf{L})| = 2$ interactions arise in these models by elimination of the $\mathsf{SU(5)}$ singlets in the $\mathsf{16}$ plet of matter after the singlet in $\mathsf{126} + \overline{\mathsf{126}}$ gains a VEV through spontaneous breaking and by elimination of the heavy Higgs fields in $\mathsf{5}+\overline{\mathsf{5}}$, $\mathsf{45}+\overline{\mathsf{45}}$ and of $\mathsf{10}+\overline{\mathsf{10}}$ plets of $\mathsf{SU(5)}$. In [30] an analysis was carried out where the heavy fields arising from $\mathsf{5}+\overline{\mathsf{5}}$, $\mathsf{45}+\overline{\mathsf{45}}$ are integrated out to get the $|\Delta(\mathsf{B}-\mathsf{L})| = 2$ interactions. In this analysis we consider the $|\Delta(\mathsf{B}-\mathsf{L})| = 2$ interactions arising from elimination of heavy $\mathsf{10}+\overline{\mathsf{10}}$ plets of $\mathsf{SU(5)}$. They arise in the decomposition of the $\mathsf{SO(10)}$ Higgs fields $\mathsf{126} + \overline{\mathsf{126}} + \mathsf{210}$ and of $\mathsf{120}$. Thus in $\mathsf{SU(5)} \times \mathsf{U(1)}$ decomposition one has a 3×3 mass matrix. The mass matrix has one zero eigenvalue which correspond to Goldstone boson corresponding to the breaking of the gauge symmetry from $\mathsf{SO(10)}$ which has 45 gauge bosons to $\mathsf{SU(5)} \times \mathsf{U(1)}$ which has only 25. Thus one combination of $\mathsf{10}+\overline{\mathsf{10}}$ which is massless is absorbed to give masses to the gauge bosons corresponding to the broken symmetry. This leaves us with two massive pairs of $\mathsf{10}+\overline{\mathsf{10}}$ which must be integrated out to generate the $|\Delta(\mathsf{B}-\mathsf{L})| = 2$ interactions. The integration gives $\mathsf{B}-\mathsf{L}$ violating five field and six field operators. The entire analysis can be directly embedded in supergravity unified models [44, 45]. One consequence of unified models is of course nucleon decay (for a broad review see [46]). One operator of interest for neutron decay is $\widehat{\mathbf{L}}\widehat{\mathbf{L}}\widehat{\mathbf{E}}^{\mathsf{c}}\widehat{\mathbf{D}}^{\mathsf{c}}\widehat{\mathbf{D}}^{\mathsf{c}}\widehat{\mathbf{U}}^{\mathsf{c}}$ which can produce purely leptons decay of the neutron, i.e., $n \to \nu l_i l_j$. One also finds the operator $\widehat{\mathbf{D}}^{\mathsf{c}}\widehat{\mathbf{D}}^{\mathsf{c}}\widehat{\mathbf{D}}^{\mathsf{c}}\widehat{\mathbf{U}}^{\mathsf{c}}\widehat{\mathbf{U}}^{\mathsf{c}}\widehat{\mathbf{U}}^{\mathsf{c}}$ which can generate $n - \bar{n}$ oscillations while variety of other operators can mediate baryogenesis.

The outline of the rest of the paper is as follows: In section 2 we give a brief description of the specific $\mathsf{SO(10)}$ missing partner model. In section 3 we discuss the spontaneous breaking of $\mathsf{SO(10)}$ to $\mathsf{SU(5)} \times \mathsf{U(1)}$ through VEV formation for the $\mathsf{SU(5)}$ singlets in $\mathsf{126} + \overline{\mathsf{126}}$ and $\mathsf{210}$. The spontaneous breaking produces a set of $(\mathsf{10}+\overline{\mathsf{10}})_{\mathsf{SU(5)}}$ of Goldstone bosons. One finds that there still remain two pairs of massive $(\mathsf{10}+\overline{\mathsf{10}})_{\mathsf{SU(5)}}$ of Higgs fields. In section 4 we show that integration over these $(\mathsf{10}+\overline{\mathsf{10}})_{\mathsf{SU(5)}}$ of Higgs fields leads to $\mathsf{B}-\mathsf{L}$ violating higher dimensional operators which are dimension 5 and dimension 6 in the superpotential. In section 5 we express these $\mathsf{B}-\mathsf{L}$ violating operators in $\mathsf{SU(3)}_C \times \mathsf{SU(2)}_L \times \mathsf{U(1)}_Y$ decomposition, i.e., in terms of the quark, lepton and Higgs fields.

2. The SO(10) Model

Consider the following $\mathsf{S0(10)}$ interactions: $\mathsf{126} \cdot \overline{\mathsf{126}}$, $(\mathsf{126} \cdot \overline{\mathsf{126}})^2$, $(\mathsf{210})^2$, $(\mathsf{210})^3$, $\mathsf{126} \cdot \overline{\mathsf{126}} \cdot \mathsf{210}$, $\mathsf{120} \cdot \mathsf{126} \cdot \mathsf{210}$ and $\mathsf{120} \cdot \overline{\mathsf{126}} \cdot \mathsf{210}$. Computation of these couplings can be carried out using the oscillator techniques [35] developed for the computation of n-point functions in [36, 37, 38, 39, 40, 41] (for related works see [42, 43]). Explicit form of the superpotential containing these interactions take the form [28]

$$
\begin{aligned}
W &= W_2 + W_3 + W_4 \\
W_2 &= m_\Phi \, \Phi_{\mu\nu\sigma\xi}\Phi_{\mu\nu\sigma\xi} + m_\Delta \, \Delta_{\mu_1\mu_2\mu_3\mu_4\mu_5} \, \overline{\Delta}_{\mu_1\mu_2\mu_3\mu_4\mu_5}
\end{aligned}
$$

$$\begin{aligned}
W_3 &= +\lambda\ \Phi_{\mu\nu\sigma\xi}\Phi_{\sigma\xi\rho\tau}\Phi_{\rho\tau\mu\nu} + \eta\ \Phi_{\mu\nu\sigma\xi}\Delta_{\mu\nu\rho\tau\zeta}\overline{\Delta}_{\sigma\xi\rho\tau\zeta} \\
&\quad + \mathrm{C}\ \Sigma_{\mu\nu\sigma}\Delta_{\nu\sigma\xi\zeta\rho}\Phi_{\mu\xi\zeta\rho} + \overline{\mathrm{C}}\ \Sigma_{\mu\nu\sigma}\overline{\Delta}_{\nu\sigma\xi\zeta\rho}\Phi_{\mu\xi\zeta\rho} \\
W_4 &= +\alpha_1\ \Delta_{\mu_1\mu_2\mu_3\mu_4\mu_5}\ \overline{\Delta}_{\mu_1\mu_2\mu_3\mu_4\mu_5}\ \Delta_{\nu_1\nu_2\nu_3\nu_4\mu_5}\ \overline{\Delta}_{\nu_1\nu_2\nu_3\nu_4\mu_5} \\
&\quad + \alpha_2\ \Delta_{\mu_1\mu_2\mu_3\mu_4\alpha}\ \overline{\Delta}_{\mu_1\mu_2\mu_3\mu_4\beta}\ \Delta_{\nu_1\nu_2\nu_3\nu_4\alpha}\ \overline{\Delta}_{\nu_1\nu_2\nu_3\nu_4\beta} \\
&\quad + \alpha_3\ \Delta_{\mu_1\mu_2\mu_3\alpha\beta}\ \overline{\Delta}_{\mu_1\mu_2\mu_3\rho\sigma}\ \Delta_{\nu_1\nu_2\nu_3\alpha\beta}\ \overline{\Delta}_{\nu_1\nu_2\nu_3\rho\sigma} \\
&\quad + \alpha_4\ \Delta_{\mu_1\mu_2\mu_3\alpha\beta}\ \overline{\Delta}_{\mu_1\mu_2\mu_3\rho\sigma}\ \Delta_{\nu_1\nu_2\nu_3\alpha\rho}\ \overline{\Delta}_{\nu_1\nu_2\nu_3\beta\sigma} \\
&\equiv W_{SSB} + W_{10} + \cdots,
\end{aligned} \tag{1}$$

Here W_{SSB} is the superpotential which leads to spontaneous breaking of $\mathsf{SO}(10)$ to $\mathsf{SU}(5)\times\mathsf{U}(1)$ and involves the $\mathsf{SU}(5)$ singlets in $\mathsf{126}+\overline{\mathsf{126}}$ and in $\mathsf{210}$ and W_{10} is the superpotential that allows one to compute the mass matrix for the $\mathsf{10}+\overline{\mathsf{10}}$ of $\mathsf{SU}(5)$ after the symmetry breaking. An explicit computation of the couplings of Eq. (1) in $\mathsf{SU}(5)\times\mathsf{U}(1)$ decomposition allows us to pull out the terms that are responsible for the breaking of $\mathsf{SO}(10)$ to $\mathsf{SU}(5)\times\mathsf{U}(1)$. As mentioned these involve the $\mathsf{SU}(5)$ singlet fields in $\mathsf{126}$ and $\overline{\mathsf{126}}$ which are $\mathbf{S}_{1_{126}}, \mathbf{S}_{1_{\overline{126}}}$ and the $\mathsf{SU}(5)$ singlet field in $\mathsf{210}$ which is $\mathbf{S}_{1_{210}}$. Thus the superpotential W_{SSB} in terms of the normalized $\mathsf{SU}(5)$ fields, is given by

$$\begin{aligned}
W_{SSB} &= 2m_\Delta\ \mathbf{S}_{1_{126}}\mathbf{S}_{1_{\overline{126}}} \\
&\quad + \frac{16}{225}\left(225\alpha_1 + \frac{45}{2}\alpha_2 + \frac{45}{16}\alpha_3\right)\ \mathbf{S}^2_{1_{126}}\mathbf{S}^2_{1_{\overline{126}}} \\
&\quad + m_\Phi\ \mathbf{S}^2_{1_{210}} \\
&\quad - \frac{1}{2\sqrt{15}}\lambda\ \mathbf{S}^3_{1_{210}} \\
&\quad - \frac{1}{\sqrt{15}}\eta\ \mathbf{S}_{1_{210}}\mathbf{S}_{1_{126}}\mathbf{S}_{1_{\overline{126}}}
\end{aligned} \tag{2}$$

Next we compute part of the superpotential of Eq. (1) that gives rise to the mass mass matrix for the $\mathsf{10}+\overline{\mathsf{10}}$ plets of $\mathsf{SU}(5)$. There are three such pairs of $\mathsf{10}+\overline{\mathsf{10}}$ plets. One pair arises from 120 plet of Higgs, a second pair from $\mathsf{210}$ and a third pair from $\mathsf{126}+\overline{\mathsf{126}}$. The superpotential involving these is given by

$$\begin{aligned}
W_{10} &= m_\Delta\ \widehat{\mathsf{H}}^{(126)ij}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij} \\
&\quad + \frac{8}{225}\left(450\alpha_1 + 45\alpha_2 + \frac{45}{8}\alpha_3\right)\mathbf{S}_{1_{126}}\mathbf{S}_{1_{\overline{126}}}\widehat{\mathsf{H}}^{(126)ij}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij} \\
&\quad + m_\Phi\ \widehat{\mathsf{H}}^{(210)ij}\ \widehat{\mathsf{H}}^{(210)}_{ij} \\
&\quad - \frac{1}{4}\sqrt{\frac{3}{5}}\lambda\ \mathbf{S}_{1_{210}}\widehat{\mathsf{H}}^{(210)ij}\ \widehat{\mathsf{H}}^{(210)}_{ij} \\
&\quad + \eta\left(-\frac{1}{5\sqrt{15}}\mathbf{S}_{1_{210}}\widehat{\mathsf{H}}^{(126)ij}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij} - \frac{1}{10}\mathbf{S}_{1_{126}}\ \widehat{\mathsf{H}}^{(210)ij}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij} - \frac{1}{10}\mathbf{S}_{1_{\overline{126}}}\ \widehat{\mathsf{H}}^{(126)ij}\ \widehat{\mathsf{H}}^{(210)}_{ij}\right) \\
&\quad + \frac{\imath\mathrm{C}}{5!}\left(-\frac{1}{4\sqrt{10}}\ \mathbf{S}_{1_{126}}\ \widehat{\mathsf{H}}^{(210)ij}\ \widehat{\mathsf{H}}^{(120)}_{ij} + \frac{1}{20}\sqrt{\frac{3}{2}}\ \mathbf{S}_{1_{210}}\ \widehat{\mathsf{H}}^{(126)ij}\ \widehat{\mathsf{H}}^{(120)}_{ij}\right) \\
&\quad + \frac{\imath\overline{\mathrm{C}}}{5!}\left(-\frac{1}{4\sqrt{10}}\ \mathbf{S}_{1_{\overline{126}}}\ \widehat{\mathsf{H}}^{(120)ij}\ \widehat{\mathsf{H}}^{(210)}_{ij} + \frac{1}{20}\sqrt{\frac{3}{2}}\ \mathbf{S}_{1_{210}}\ \widehat{\mathsf{H}}^{(120)ij}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij}\right)
\end{aligned} \tag{3}$$

To compute the mass matrix for the $\mathsf{SU}(5)$ $\mathsf{10}+\overline{\mathsf{10}}$ plets we first need to break the $\mathsf{SO}(10)$ symmetry to $SU5)\times\mathsf{U}(1)$. This is discussed next.

3. SO(10) symmetry breaking to SU(5) × U(1)

Spontaneous breaking of SO(10) to SU(5) × U(1) is achieved by the following two constraints obtained by the variation of Eq. (2) with $<\mathbf{S}_{1_{210}}>$ and $<\mathbf{S}_{1_{\overline{126}}}>$ so that

$$\frac{\partial < W_{SSB} >}{\partial < \mathbf{S}_{1_{210}} >} = 0 \qquad \Rightarrow 2m_\Phi < \mathbf{S}_{1_{210}} > -\frac{1}{2}\sqrt{\frac{3}{5}}\lambda < \mathbf{S}_{1_{210}} >^2 - \frac{1}{\sqrt{15}}\eta < \mathbf{S}_{1_{126}} >< \mathbf{S}_{1_{\overline{126}}} >= 0, \quad (4)$$

$$\frac{\partial < W_{SSB} >}{\partial < \mathbf{S}_{1_{\overline{126}}} >} = 0 \qquad \Rightarrow 2m_\Delta < \mathbf{S}_{1_{126}} > -\frac{1}{\sqrt{15}}\eta < \mathbf{S}_{1_{210}} >< \mathbf{S}_{1_{126}} > +\frac{2}{5}k < \mathbf{S}_{1_{126}} >^2 < \mathbf{S}_{1_{\overline{126}}} >= 0 \quad (5)$$

where

$$k \equiv 20\alpha_1 + 2\alpha_2 + \alpha_3, \qquad (6)$$

Elimination of $<\mathbf{S}_{1_{210}}>$ between the Eqs. (4) and (5) leads to a quadratic equation to determine $<\mathbf{S}_{1_{126}}><\mathbf{S}_{1_{\overline{126}}}>$ so that

$$18\lambda k^2 < \mathbf{S}_{1_{\overline{126}}} >^2 < \mathbf{S}_{1_{126}} >^2 + \left(5\eta^3 + 180\lambda k m_\Delta - 60\eta k m_\Phi\right) < \mathbf{S}_{1_{\overline{126}}} >< \mathbf{S}_{1_{126}} > \\ +450\lambda m_\Delta^2 - 300\eta m_\Delta m_\Phi = 0 \qquad (7)$$

Once $<\mathbf{S}_{1_{126}}><\mathbf{S}_{1_{\overline{126}}}>$ is determined, $<\mathbf{S}_{1_{210}}>$ can be found from

$$< \mathbf{S}_{1_{210}} >= \frac{2}{\eta}\sqrt{\frac{3}{5}}\left(5m_\Delta + k < \mathbf{S}_{1_{\overline{126}}} >< \mathbf{S}_{1_{\overline{126}}} >\right) \qquad (8)$$

We now determine the masses of the SU(5) singlets after spontaneous symmetry breaking and in particular identify the Goldstone Boson corresponding to the breaking of B − L symmetry. To that end, we expand the singlet fields in Eq. (2) about the VEVs obtained through Eqs. (7) and (8):

$$\begin{aligned} \mathbf{S}_{1_{210}} &= \mathbf{S}'_{1_{210}} + < \mathbf{S}_{1_{210}} > \\ \mathbf{S}_{1_{126}} &= \mathbf{S}'_{1_{126}} + < \mathbf{S}_{1_{126}} > \\ \mathbf{S}_{1_{\overline{126}}} &= \mathbf{S}'_{1_{\overline{126}}} + < \mathbf{S}_{1_{\overline{126}}} > \end{aligned} \qquad (9)$$

We obtain

$$\begin{aligned} W_{SSB} = &\left(m_\Phi - \frac{1}{2}\sqrt{\frac{3}{5}}\lambda < \mathbf{S}_{1_{210}} >\right)\mathbf{S}'^2_{1_{210}} \\ &+ \left(\frac{1}{5}k < \mathbf{S}_{1_{\overline{126}}} >^2\right)\mathbf{S}'^2_{1_{126}} \\ &+ \left(\frac{1}{5}k < \mathbf{S}_{1_{126}} >^2\right)\mathbf{S}'^2_{1_{\overline{126}}} \\ &+ \left(2m_\Delta - \frac{1}{\sqrt{15}}\eta < \mathbf{S}_{1_{210}} > + \frac{4}{5}k < \mathbf{S}_{1_{126}} >< \mathbf{S}_{1_{\overline{126}}} >\right)\mathbf{S}'_{1_{126}}\mathbf{S}'_{1_{\overline{126}}} \\ &+ \left(-\frac{1}{\sqrt{15}}\eta < \mathbf{S}_{1_{\overline{126}}} >\right)\mathbf{S}'_{1_{210}}\mathbf{S}'_{1_{126}} \\ &+ \left(-\frac{1}{\sqrt{15}}\eta < \mathbf{S}_{1_{126}} >\right)\mathbf{S}'_{1_{210}}\mathbf{S}'_{1_{\overline{126}}} \\ &+ \cdots \end{aligned} \qquad (10)$$

Note that $\left(2m_\Delta - \frac{1}{\sqrt{15}}\eta < \mathbf{S}_{1_{210}} > + \frac{4}{5}k < \mathbf{S}_{1_{126}} >< \mathbf{S}_{1_{\overline{126}}} >\right)\Big|_{\text{Eq.(8)}} = \frac{2}{5}k < \mathbf{S}_{1_{126}} >$ $\times < \mathbf{S}_{1_{\overline{126}}} >$.
Writing

$$W_{SSB} = \left(\mathbf{S}'_{1_{210}},\ \mathbf{S}'_{1_{126}},\ \mathbf{S}'_{1_{\overline{126}}}\right) M_{\text{singlet}} \begin{pmatrix} \mathbf{S}'_{1_{210}} \\ \mathbf{S}'_{1_{126}} \\ \mathbf{S}'_{1_{\overline{126}}} \end{pmatrix} + \cdots \tag{11}$$

where

$$M_{\text{singlet}} = \begin{pmatrix} m_\Phi - \frac{1}{2}\sqrt{\frac{3}{5}}\lambda < \mathbf{S}_{1_{210}} > & -\frac{1}{2\sqrt{15}}\eta < \mathbf{S}_{1_{\overline{126}}} > & -\frac{1}{2\sqrt{15}}\eta < \mathbf{S}_{1_{126}} > \\ -\frac{1}{2\sqrt{15}}\eta < \mathbf{S}_{1_{\overline{126}}} > & \frac{1}{5}k < \mathbf{S}_{1_{\overline{126}}} >^2 & \frac{1}{5}k < \mathbf{S}_{1_{126}} >< \mathbf{S}_{1_{\overline{126}}} > \\ -\frac{1}{2\sqrt{15}}\eta < \mathbf{S}_{1_{126}} > & \frac{1}{5}k < \mathbf{S}_{1_{126}} >< \mathbf{S}_{1_{\overline{126}}} > & \frac{1}{5}k < \mathbf{S}_{1_{126}} >^2 \end{pmatrix} \tag{12}$$

It is then easily checked that det $\left(M_{\text{singlet}}\right) = 0$. This zero eigenmode is the Goldstone Boson corresponding to the breaking of $\mathsf{B}-\mathsf{L}$ symmetry.

3.1. 10*-plet Higgs mass matrix*

Next we compute the mass matrix of 10 plets of SU(5) due to mixing between $\mathsf{10}_{\mathsf{120}}$, $\overline{\mathsf{10}}_{\mathsf{120}}$, $\mathsf{10}_{\mathsf{126}}$, $\overline{\mathsf{10}}_{\overline{\mathsf{126}}}$, $\mathsf{10}_{\mathsf{210}}$ and $\overline{\mathsf{10}}_{\mathsf{210}}$. Collecting the terms in W_{10} we get

$$\begin{aligned} W_{10} &= \left(m_\Phi - \frac{1}{4}\sqrt{\frac{3}{5}}\lambda < \mathbf{S}_{1_{210}} >\right) \widehat{\mathsf{H}}^{(210)ij}\ \widehat{\mathsf{H}}^{(210)}_{ij} \\ &+ \left(m_\Delta - \frac{1}{5\sqrt{15}}\eta < \mathbf{S}_{1_{210}} > + \frac{1}{5}k < \mathbf{S}_{1_{126}} >< \mathbf{S}_{1_{\overline{126}}} >\right) \widehat{\mathsf{H}}^{(126)ij}\widehat{\mathsf{H}}^{(\overline{126})}_{ij} \\ &+ \left(-\frac{1}{10}\eta < \mathbf{S}_{1_{126}} >\right) \widehat{\mathsf{H}}^{(210)ij}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij} \\ &+ \left(-\frac{1}{10}\eta < \mathbf{S}_{1_{\overline{126}}} >\right)\ \widehat{\mathsf{H}}^{(126)ij}\ \widehat{\mathsf{H}}^{(210)}_{ij} \\ &+ \left(-\frac{1}{4\sqrt{10}}c < \mathbf{S}_{1_{126}} >\right) \widehat{\mathsf{H}}^{(210)ij}\ \widehat{\mathsf{H}}^{(120)}_{ij} \\ &+ \left(-\frac{1}{4\sqrt{10}}\bar{c} < \mathbf{S}_{1_{\overline{126}}} >\right) \widehat{\mathsf{H}}^{(120)ij}\ \widehat{\mathsf{H}}^{(210)}_{ij} \\ &+ \left(\frac{1}{20}\sqrt{\frac{3}{2}}c < \mathbf{S}_{1_{210}} >\right) \widehat{\mathsf{H}}^{(126)ij}\ \widehat{\mathsf{H}}^{(120)}_{ij} \\ &+ \left(\frac{1}{20}\sqrt{\frac{3}{2}}\bar{c} < \mathbf{S}_{1_{210}} >\right) \widehat{\mathsf{H}}^{(120)ij}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij} \\ &\equiv \left(\widehat{\mathsf{H}}^{(120)}_{ij}, \widehat{\mathsf{H}}^{(\overline{126})}_{ij}, \widehat{\mathsf{H}}^{(210)}_{ij}\right) M_{10} \begin{pmatrix} \widehat{\mathsf{H}}^{(120)ij} \\ \widehat{\mathsf{H}}^{(126)ij} \\ \widehat{\mathsf{H}}^{(210)ij} \end{pmatrix} \end{aligned} \tag{13}$$

where

$$M_{10}=\begin{pmatrix} 0 & \frac{1}{20}\sqrt{\frac{3}{2}}\bar{c}<\mathbf{S}_{1_{210}}> & -\frac{1}{4\sqrt{10}}\bar{c}<\mathbf{S}_{1_{\overline{126}}}> \\ \frac{1}{20}\sqrt{\frac{3}{2}}c<\mathbf{S}_{1_{210}}> & m_\Delta-\frac{1}{5\sqrt{15}}\eta<\mathbf{S}_{1_{210}}>+\frac{1}{5}k<\mathbf{S}_{1_{126}}><\mathbf{S}_{1_{\overline{126}}}> & -\frac{1}{10}\eta<\mathbf{S}_{1_{\overline{126}}}> \\ -\frac{1}{4\sqrt{10}}c<\mathbf{S}_{1_{126}}> & -\frac{1}{10}\eta<\mathbf{S}_{1_{126}}> & m_\Phi-\frac{1}{4}\sqrt{\frac{3}{5}}\lambda<\mathbf{S}_{1_{210}}> \end{pmatrix} \quad (14)$$

and

$$c\equiv\frac{\imath \mathrm{C}}{5!},\qquad \bar{c}\equiv\frac{\imath\overline{\mathrm{C}}}{5!} \quad (15)$$

$$\begin{aligned} \det(M_{10}) &= -\frac{1}{48000}c\bar{c}\Big[180m_\Phi<\mathbf{S}_{1_{210}}>^2-9\sqrt{15}\lambda<\mathbf{S}_{1_{210}}>^3-16\sqrt{15}\eta<\mathbf{S}_{1_{210}}><\mathbf{S}_{1_{126}}> \\ &\quad\times<\mathbf{S}_{1_{\overline{126}}}>+60<\mathbf{S}_{1_{126}}><\mathbf{S}_{1_{\overline{126}}}>\Big(5m_\Delta+k<\mathbf{S}_{1_{126}}><\mathbf{S}_{1_{\overline{126}}}>\Big)\Big] \\ \det(M_{10})|_{\text{Eq.(8)}} &= \frac{3}{20000}\frac{c\bar{c}}{\eta^3}\Big(5m_\Delta+k<\mathbf{S}_{1_{126}}><\mathbf{S}_{1_{\overline{126}}}>\Big)\Big[18\lambda k^2<\mathbf{S}_{1_{\overline{126}}}>^2<\mathbf{S}_{1_{126}}>^2 \\ &\quad+\Big(5\eta^3+180\lambda k m_\Delta-60\eta k m_\Phi\Big)<\mathbf{S}_{1_{\overline{126}}}><\mathbf{S}_{1_{126}}>+450\lambda m_\Delta^2-300\eta m_\Delta m_\Phi\Big] \\ \det(M_{10})|_{\text{Eq.(7)}} &= 0 \end{aligned}$$

The non-zero eigenvalues (Λ's) of the 10-plet mass matrix satisfies the following quadratic equation

$$\Lambda^2-x\Lambda+y=0 \quad (16)$$

where x and y are given by

$$\begin{aligned} x&=\frac{3}{50}\Big(5m_\Delta+k<\mathbf{S}_{1_{126}}><\mathbf{S}_{1_{\overline{126}}}>\Big)\Big(2-5\frac{\lambda}{\eta}\Big)+m_\Phi \\ y&=\frac{c\bar{c}}{800}\left[\Big(-5+2\frac{\eta}{\lambda}-24\frac{km_\Phi}{\lambda\eta}\Big)<\mathbf{S}_{1_{126}}><\mathbf{S}_{1_{\overline{126}}}>-120\frac{m_\Delta m_\Phi}{\lambda\eta}\right]. \end{aligned} \quad (17)$$

and where we have used the spontaneous symmetry breaking constraints of Eq. (3) and (8) in obtaining Eq. (16) and Eq. (17). From Eq. (16) we note that for the case when $y=0$, one finds that there is another eigenvalue which also vanishes. This means that a technically natural large hierarchy can exist between the two eigenvalues of the 10-plet of the massive $10+\overline{10}$ Higgs fields.

4. $\mathsf{B}-\mathsf{L}=-2$ operators from $(\mathbf{10}+\overline{\mathbf{10}})_{\mathsf{SU(5)}}$ Higgs mediation

In this section we compute the $\mathsf{B}-\mathsf{L}=-2$ interactions arising from the elimination of 10_{120}, $\overline{10}_{120}$ and $\overline{10}_{\overline{126}}$ appearing in the matter-Higgs interactions. The $\mathsf{B}-\mathsf{L}$ violating interactions arise as a consequence of the singlets of 126 and $\overline{126}$ gaining VEVs. In turn this VEV formation gives mass to the singlets of the 16-plets of matter. Thus the heavy fields in the model after spontaneous breaking of the GUT symmetry consists of all of the Higgs fields except for a pair of light Higgs doublets and in addition the singlet fields arising from the 16-plets of matter. From the couplings of Higgs with matter we are interested in pulling out only the parts that give

$\mathsf{B}-\mathsf{L}=-2$. To obtain a low energy effective Lagrangian which contains $\mathsf{B}-\mathsf{L}=-2$ violations, we integrate on all the relevant heavy fields which can generate such interactions. These include $\mathsf{SU}(5)$ fields $\mathsf{5}+\overline{\mathsf{5}}$, $\mathsf{10}+\overline{\mathsf{10}}$, and $\mathsf{45}+\overline{\mathsf{45}}$ (except for the light Higgses) and the matter singlets. The integration on $\mathsf{5}+\overline{\mathsf{5}}$ and $\mathsf{45}+\overline{\mathsf{45}}$ has already been done in our previous paper [30]. We use the following integration path of the heavy fields: First we integrate on the $\mathsf{10}+\overline{\mathsf{10}}$ plet of Higgs since they are all superheavy keeping in mind that there is mixing among the $\mathsf{10}+\overline{\mathsf{10}}$ arising from the $\mathsf{120}$, $\mathsf{126}+\overline{\mathsf{126}}$ and $\mathsf{210}$. Next we integrate on the matter singlets and finally integrate on the remaining heavy Higgs fields. Any other integration path gives the same result.

We begin by displaying the cubic matter-Higgs couplings which consists of $\mathsf{16}\cdot\mathsf{16}\cdot\mathsf{10}$, $\mathsf{16}\cdot\mathsf{16}\cdot\mathsf{120}$ and $\mathsf{16}\cdot\mathsf{16}\cdot\overline{\mathsf{126}}$ couplings. In $\mathsf{SU}(5)$ decomposition they are given by [36]

$$W^{(\mathsf{16}\cdot\mathsf{16}\cdot\mathsf{10})} = \imath 2\sqrt{2} f^{(10_r+)}_{\acute{x}\acute{y}} \left(\widehat{\mathsf{M}}^{ij}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}i}\ \widehat{\mathsf{H}}^{(10_r)}_{j} - \widehat{\mathsf{M}}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}i}\ \widehat{\mathsf{H}}^{(10_r)i} + \frac{1}{8}\epsilon_{ijklm}\ \widehat{\mathsf{M}}^{ij}_{\acute{x}}\ \widehat{\mathsf{M}}^{kl}_{\acute{y}}\ \widehat{\mathsf{H}}^{(10_r)m} \right), \tag{18}$$

$$\begin{aligned} W^{(\mathsf{16}\cdot\mathsf{16}\cdot\mathsf{120})} &= \imath \frac{2}{\sqrt{3}} f^{(120-)}_{\acute{x}\acute{y}} \Big(2\widehat{\mathsf{M}}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}i}\ \widehat{\mathsf{H}}^{(120)i} + \widehat{\mathsf{M}}^{ij}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}}\ \widehat{\mathsf{H}}^{(120)}_{ij} + \widehat{\mathsf{M}}_{\acute{x}i}\ \widehat{\mathsf{M}}_{\acute{y}j}\ \widehat{\mathsf{H}}^{(120)ij} \\ &\quad - \widehat{\mathsf{M}}^{ij}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}i}\ \widehat{\mathsf{H}}^{(120)}_{j} + \widehat{\mathsf{M}}_{\acute{x}i}\ \widehat{\mathsf{M}}^{jk}_{\acute{y}}\ \widehat{\mathsf{H}}^{(120)i}_{jk} - \frac{1}{4}\epsilon_{ijklm}\ \widehat{\mathsf{M}}^{ij}_{\acute{x}}\ \widehat{\mathsf{M}}^{mn}_{\acute{y}}\ \widehat{\mathsf{H}}^{(120)kl}_{n} \Big), \end{aligned} \tag{19}$$

$$\begin{aligned} W^{(\mathsf{16}\cdot\mathsf{16}\cdot\overline{\mathsf{126}})} &= \imath\sqrt{\frac{2}{15}} f^{(\overline{126}+)}_{\acute{x}\acute{y}} \Big(-\sqrt{2}\widehat{\mathsf{M}}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}}\ \widehat{\mathsf{H}}^{(\overline{126})} - \sqrt{3}\widehat{\mathsf{M}}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}i}\ \widehat{\mathsf{H}}^{(\overline{126})i} + \widehat{\mathsf{M}}_{\acute{x}}\ \widehat{\mathsf{M}}^{ij}_{\acute{y}}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij} \\ &\quad - \frac{1}{8\sqrt{3}}\epsilon_{ijklm}\ \widehat{\mathsf{M}}^{ij}_{\acute{x}}\ \widehat{\mathsf{M}}^{kl}_{\acute{y}}\ \widehat{\mathsf{H}}^{(\overline{126})m} - \widehat{\mathsf{M}}_{\acute{x}i}\ \widehat{\mathsf{M}}_{\acute{y}j}\ \widehat{\mathsf{H}}^{(\overline{126})ij}_{(S)} + \widehat{\mathsf{M}}^{ij}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(\overline{126})k}_{ij} \\ &\quad - \frac{1}{12\sqrt{2}}\epsilon_{ijklm}\ \widehat{\mathsf{M}}^{lm}_{\acute{x}}\ \widehat{\mathsf{M}}^{rs}_{\acute{y}}\ \widehat{\mathsf{H}}^{(\overline{126})ijk}_{rs} \Big), \end{aligned} \tag{20}$$

where the front factors $f^{(\cdot\pm)}_{\acute{x}\acute{y}}$ in Eqs. (18) - (20) exhibit the symmetry and anti-symmetry in the generation indices: $f^{(\cdot\pm)}_{\acute{x}\acute{y}} = \frac{1}{2}\left(f^{(\cdot)}_{\acute{x}\acute{y}} \pm f^{(\cdot)}_{\acute{y}\acute{x}}\right)$.

The Higgs $\mathsf{10}$ plets of $\mathsf{SU}(5)$ appearing in the trilinear matter couplings are superheavy and their decays violate $\mathsf{B}-\mathsf{L}$. We need to eliminate $\mathsf{10}_{\mathsf{120}}$, $\overline{\mathsf{10}}_{\mathsf{120}}$ and $\overline{\mathsf{10}}_{\overline{\mathsf{126}}}$ using the mixing mass matrix M_{10}. These arise only from the $\mathsf{120}$ and $\overline{\mathsf{126}}$ Higgs couplings to matter while the $\mathsf{10}$ plet Higgs mass matrix arises from the Higgs sectors $\mathsf{120}$, $\mathsf{126}$, $\mathsf{210}$. The relevant superpotential takes the form

$$\begin{aligned} W' &= \imath\frac{2}{\sqrt{3}} f^{(120-)}_{\acute{x}\acute{y}}\ \widehat{\mathsf{M}}^{ij}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}}\ \widehat{\mathsf{H}}^{(120)}_{ij} + \imath\frac{2}{\sqrt{3}} f^{(120-)}_{\acute{x}\acute{y}}\ \widehat{\mathsf{M}}_{\acute{x}i}\ \widehat{\mathsf{M}}_{\acute{y}j}\ \widehat{\mathsf{H}}^{(120)ij} + \imath\sqrt{\frac{2}{15}} f^{(\overline{126}+)}_{\acute{x}\acute{y}}\ \widehat{\mathsf{M}}_{\acute{x}}\ \widehat{\mathsf{M}}^{ij}_{\acute{y}}\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij} \\ &\quad + \left(\widehat{\mathsf{H}}^{(120)}_{ij},\ \widehat{\mathsf{H}}^{(\overline{126})}_{ij},\ \widehat{\mathsf{H}}^{(210)}_{ij} \right) M_{10} \begin{pmatrix} \widehat{\mathsf{H}}^{(120)ij} \\ \widehat{\mathsf{H}}^{(126)ij} \\ \widehat{\mathsf{H}}^{(210)ij} \end{pmatrix} \end{aligned} \tag{21}$$

The mass matrix M_{10} has a zero eigenvalue and is a Goldstone mode which is absorbed by the $\mathsf{10}+\overline{\mathsf{10}}$ plet of $\mathsf{SU}(5)$ of the vector fields in the adjoint representation $\mathsf{45}$ plet of $\mathsf{SO}(10)$ to make them heavy through the Higgs mechanism. It is necessary to remove this zero eigenmode before the elimination of the $\mathsf{10}+\overline{\mathsf{10}}$ fields in Eq. (21). To that end, we must diagonalize the mass matrix M_{10} by a biunitary transformation:

$$U^{\dagger} M_{10} \mathsf{V} = \mathrm{diag}\,(m_1, m_2, 0) \tag{22}$$

and where the columns of the 3×3 unitary matrices U and V represent the eigenvectors of matrices $M_{10}^{\dagger} M_{10}$ and $M_{10} M_{10}^{\dagger}$, respectively:

$$U^{\dagger}[M_{10}^{\dagger} M_{10}]U = V^{\dagger}[M_{10} M_{10}^{\dagger}]V = \text{diag}\left(m_1^2, m_2^2, 0\right) \tag{23}$$

We now recast Eq. (21) in the following form:

$$W' = \sum_{p,q=1}^{2} \sum_{o=1}^{3} \left[\overline{F}_p \left(M_D\right)_{pq} F_q + \overline{F}_q U_{qo}^{\dagger} \overline{J}_o + J_o V_{oq} F_q\right] \tag{24}$$

where

$$\begin{aligned}
\overline{F} &= \left(\widehat{\mathsf{H}}_{ij}^{(120)},\ \widehat{\mathsf{H}}_{ij}^{(\overline{126})},\ \widehat{\mathsf{H}}_{ij}^{(210)}\right) U, \\
F &= V^{\dagger} \begin{pmatrix} \widehat{\mathsf{H}}^{(120)ij} \\ \widehat{\mathsf{H}}^{(126)ij} \\ \widehat{\mathsf{H}}^{(210)ij} \end{pmatrix}, \\
\overline{J} &= \begin{pmatrix} \imath \frac{2}{\sqrt{3}} f_{\acute{x}\acute{y}}^{(120-)}\ \widehat{\mathsf{M}}_{\acute{x}}^{ij}\ \widehat{\mathsf{M}}_{\acute{y}} \\ \imath \sqrt{\frac{2}{15}} f_{\acute{x}\acute{y}}^{(\overline{126}+)}\ \widehat{\mathsf{M}}_{\acute{x}}\ \widehat{\mathsf{M}}_{\acute{y}}^{ij} \\ 0 \end{pmatrix} \\
J &= \left(\imath \frac{2}{\sqrt{3}} f_{\acute{x}\acute{y}}^{(120-)}\ \widehat{\mathsf{M}}_{\acute{x}i}\ \widehat{\mathsf{M}}_{\acute{y}j},\ 0,\ 0\right)
\end{aligned} \tag{25}$$

On using $\frac{\partial W'}{\partial \overline{F}_p} = 0$ and $\frac{\partial W'}{\partial F_p} = 0$, we get

$$W' = \frac{2}{3} f_{\acute{w}\acute{x}}^{(120-)} \left[2\alpha f_{\acute{y}\acute{z}}^{(120-)} + \sqrt{\frac{2}{5}} \beta f_{\acute{y}\acute{z}}^{(\overline{126}+)}\right] \widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}}^{ij}\ \widehat{\mathsf{M}}_{\acute{z}}, \tag{26}$$

where

$$\begin{aligned}
\alpha &\equiv \frac{V_{11} U_{11}^{\dagger}}{m_1} + \frac{V_{12} U_{21}^{\dagger}}{m_2}, \\
\beta &\equiv \frac{V_{11} U_{12}^{\dagger}}{m_1} + \frac{V_{12} U_{22}^{\dagger}}{m_2}.
\end{aligned} \tag{27}$$

Next we assume that because of spontaneous symmetry breaking the singlet field in the $\overline{126}$-plet of Higgs field develops a VEV, i.e., $\widehat{\mathsf{H}}^{(\overline{126})} \equiv < \mathbf{S}_{1_{\overline{126}}} > \neq 0$, which gives mass to the singlets in the 16-plet of matter fields. Collecting the terms which contain the singlet fields of matter from Eqs. (18) - (20) and Eq. (26) we have

$$\begin{aligned}
W &= \frac{2}{3} f_{\acute{w}\acute{x}}^{(120-)} \left[2\alpha f_{\acute{y}\acute{z}}^{(120-)} + \sqrt{\frac{2}{5}} \beta f_{\acute{y}\acute{z}}^{(\overline{126}+)}\right] \widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}}^{ij}\ \widehat{\mathsf{M}}_{\acute{z}} \\
&+ \widehat{\mathsf{M}}_{\acute{x}} \left\{ -\imath 2\sqrt{2} f_{\acute{x}\acute{y}}^{(10_r+)} \widehat{\mathsf{M}}_{\acute{y}i}\ \widehat{\mathsf{H}}^{(10_r)i} + \imath \frac{4}{\sqrt{3}} f_{\acute{x}\acute{y}}^{(120-)} \widehat{\mathsf{M}}_{\acute{y}i}\ \widehat{\mathsf{H}}^{(120)i} \right. \\
&\left. - \imath \sqrt{\frac{2}{5}} f_{\acute{x}\acute{y}}^{(\overline{126}+)} \widehat{\mathsf{M}}_{\acute{y}i}\ \widehat{\mathsf{H}}^{(\overline{126})i} \right\} + \frac{1}{2} \widehat{\mathsf{M}}_{\acute{x}} \left\{ -\imath \frac{4}{\sqrt{15}} f_{\acute{x}\acute{y}}^{(\overline{126}+)} < \mathbf{S}_{1_{\overline{126}}} > \right\} \widehat{\mathsf{M}}_{\acute{y}}
\end{aligned} \tag{28}$$

In the above equation the mass term for 1_{16} violates $\mathsf{B}-\mathsf{L}$. Next eliminating $\widehat{\mathsf{M}}_{\acute{x}}$ through $\dfrac{\partial W}{\partial \widehat{\mathsf{M}}_{\acute{x}}}=0$, we get the following interactions:

$$\begin{aligned} W \;=\;& \mathcal{A}_{\acute{u}\acute{v},\acute{w}\acute{x},\acute{y}\acute{z}}\ \widehat{\mathsf{M}}_{\acute{u}i}\ \widehat{\mathsf{M}}_{\acute{v}j}\ \widehat{\mathsf{M}}_{\acute{w}k}\ \widehat{\mathsf{M}}_{\acute{x}l}\ \widehat{\mathsf{M}}^{ij}_{\acute{y}}\ \widehat{\mathsf{M}}^{kl}_{\acute{z}} \\ &+ \mathcal{B}^{(r)}_{\acute{w}\acute{x},\acute{y}\acute{z}}\ \widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(10_r)k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}} \\ &+ \mathcal{C}_{\acute{w}\acute{x},\acute{y}\acute{z}}\ \widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(120)k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}} \\ &+ \mathcal{D}_{\acute{w}\acute{x},\acute{y}\acute{z}}\ \widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(\overline{126})k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}} \end{aligned} \tag{29}$$

where

$$\begin{aligned} \mathcal{A}_{\acute{u}\acute{v},\acute{w}\acute{x},\acute{y}\acute{z}} &= \frac{1}{<\mathbf{S}_{1_{\overline{126}}}>}\left(-\frac{\imath}{3}\sqrt{\frac{5}{3}}\right) f^{(120-)}_{\acute{u}\acute{v}} f^{(120-)}_{\acute{w}\acute{x}} \left\{-2\alpha^2\left[f^{(120-)}f^{(\overline{126}+)^{-1}}f^{(120-)}\right]+\frac{1}{5}\beta^2 f^{(\overline{126}+)}\right\}_{\acute{y}\acute{z}}, \\ \mathcal{B}^{(r)}_{\acute{w}\acute{x},\acute{y}\acute{z}} &= \frac{1}{<\mathbf{S}_{1_{\overline{126}}}>}\left(-\sqrt{\frac{10}{3}}\right) f^{(120-)}_{\acute{w}\acute{x}} \left\{f^{(10_r+)}f^{(\overline{126}+)^{-1}}\left[-2\alpha f^{(120-)}+\sqrt{\frac{2}{5}}\beta f^{(\overline{126}+)}\right]\right\}_{\acute{y}\acute{z}}, \\ \mathcal{C}_{\acute{w}\acute{x},\acute{y}\acute{z}} &= \frac{1}{<\mathbf{S}_{1_{\overline{126}}}>}\left(-\frac{2\sqrt{5}}{3}\right) f^{(120-)}_{\acute{w}\acute{x}} \left\{f^{(120-)}f^{(\overline{126}+)^{-1}}\left[-2\alpha f^{(120-)}+\sqrt{\frac{2}{5}}\beta f^{(\overline{126}+)}\right]\right\}_{\acute{y}\acute{z}}, \\ \mathcal{D}_{\acute{w}\acute{x},\acute{y}\acute{z}} &= \frac{1}{<\mathbf{S}_{1_{\overline{126}}}>}\left(-\frac{1}{\sqrt{6}}\right) f^{(120-)}_{\acute{w}\acute{x}} \left[-2\alpha f^{(120-)}+\sqrt{\frac{2}{5}}\beta f^{(\overline{126}+)}\right]_{\acute{y}\acute{z}}. \end{aligned} \tag{30}$$

5. $\mathsf{B}-\mathsf{L}=-2$ operators in terms of quark, lepton and Higgs fields

In this section we compute the operators as given by Eq. (29).

5.1. *Operators arising from* $\widehat{\mathsf{M}}_{\acute{u}i}\ \widehat{\mathsf{M}}_{\acute{v}j}\ \widehat{\mathsf{M}}_{\acute{w}k}\ \widehat{\mathsf{M}}_{\acute{x}l}\ \widehat{\mathsf{M}}^{ij}_{\acute{y}}\ \widehat{\mathsf{M}}^{kl}_{\acute{z}}$

$$\begin{aligned} W_{\mathcal{A}} \;\equiv\;& \mathcal{A}_{\acute{u}\acute{v},\acute{w}\acute{x},\acute{y}\acute{z}}\ \widehat{\mathsf{M}}_{\acute{u}i}\ \widehat{\mathsf{M}}_{\acute{v}j}\ \widehat{\mathsf{M}}_{\acute{w}k}\ \widehat{\mathsf{M}}_{\acute{x}l}\ \widehat{\mathsf{M}}^{ij}_{\acute{y}}\ \widehat{\mathsf{M}}^{kl}_{\acute{z}} \\ =\;& \mathcal{A}_{\acute{u}\acute{v},\acute{w}\acute{x},\acute{y}\acute{z}}\Big[\epsilon^{\alpha\beta\gamma}\epsilon^{\rho\sigma\lambda}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{u}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\rho}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\sigma}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{y}\gamma}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\lambda}+\epsilon^{ab}\epsilon^{cd}\ \widehat{\mathbf{L}}_{\acute{u}a}\ \widehat{\mathbf{L}}_{\acute{v}b}\ \widehat{\mathbf{L}}_{\acute{w}c}\ \widehat{\mathbf{L}}_{\acute{x}d}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{y}}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} \\ &+2\epsilon^{\alpha\beta\gamma}\epsilon^{ab}\ \widehat{\mathbf{L}}_{\acute{u}a}\ \widehat{\mathbf{L}}_{\acute{v}b}\widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{y}}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\gamma}+4\epsilon^{\alpha\beta\gamma}\ \widehat{\mathbf{L}}_{\acute{u}a}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\rho}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\ \widehat{\mathbf{Q}}^{a\rho}_{\acute{y}}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\gamma} \\ &+4\epsilon^{ab}\ \widehat{\mathbf{L}}_{\acute{u}a}\ \widehat{\mathbf{L}}_{\acute{v}b}\ \widehat{\mathbf{L}}_{\acute{w}c}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\alpha}\widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{y}}\ \widehat{\mathbf{Q}}^{c\alpha}_{\acute{z}}+4\ \widehat{\mathbf{L}}_{\acute{u}a}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\alpha}\ \widehat{\mathbf{L}}_{\acute{w}b}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\ \widehat{\mathbf{Q}}^{a\alpha}_{\acute{y}}\ \widehat{\mathbf{Q}}^{b\beta}_{\acute{z}}\Big] \end{aligned} \tag{31}$$

5.2. *Operators arising from* $\widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(10_r)k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}}$

$$W_{\mathcal{B}}=\mathcal{B}^{(r)}_{\acute{w}\acute{x},\acute{y}\acute{z}}\ \widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(10_r)k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}}, \tag{32}$$

(i) **Evaluating $\widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}a}\ \widehat{\mathsf{H}}^{(10_r)a}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}}$**

Here we have couplings involving the 10-plet of Higgs doublets fields $\widehat{\mathsf{H}}^{(10_r)a}$ $(r=1,2)$ each of which contains one light Higgs doublet and six heavy Higgs doublets. The light Higgs doublet leads to operators involving five fields four of which are matter fields. The heavy Higgs doublet fields must be eliminated and their elimination gives rise to operators involving six fields all of which are matter fields. These are all $\mathsf{B}-\mathsf{L}=-2$ operators. In addition elimination of heavy Higgs doublets also lead to $\mathsf{B}-\mathsf{L}=0$ operators involving four matter fields. Thus together we get the following set of operators

$$\begin{aligned} W_{\mathcal{B}}^{(Doublet)} = & \sum_{r=1}^{2} U_{d_{r1}} \mathcal{B}^{(r)}_{\acute{w}\acute{x},\acute{y}\acute{z}} \Big[\epsilon^{\alpha\beta\gamma}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\alpha}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\, \widehat{\mathbf{L}}_{\acute{y}a}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\gamma}\, \widehat{\mathbf{H}}^{a}_{\mathbf{u}} + \epsilon^{ab}\, \widehat{\mathbf{L}}_{\acute{w}a}\, \widehat{\mathbf{L}}_{\acute{x}b}\, \widehat{\mathbf{L}}_{\acute{y}c}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}}\, \widehat{\mathbf{H}}^{c}_{\mathbf{u}} \\ & +2\, \widehat{\mathbf{L}}_{\acute{w}a}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\alpha}\, \widehat{\mathbf{L}}_{\acute{y}b}\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{z}}\, \widehat{\mathbf{H}}^{b}_{\mathbf{u}} \Big] \\ & -\imath 4\sqrt{2} \Big[\epsilon^{\alpha\beta\gamma}\epsilon^{ab}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\alpha}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\, \widehat{\mathbf{L}}_{\acute{y}b}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\gamma} - \epsilon^{\beta\gamma\rho}\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\alpha}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\beta}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\gamma}\, \widehat{\mathbf{L}}_{\acute{y}a}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\rho} \\ & +\epsilon^{ab}\epsilon^{cd}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{L}}_{\acute{w}c}\, \widehat{\mathbf{L}}_{\acute{x}d}\, \widehat{\mathbf{L}}_{\acute{y}b}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} - \epsilon^{bc}\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\alpha}\, \widehat{\mathbf{L}}_{\acute{w}b}\, \widehat{\mathbf{L}}_{\acute{x}c}\, \widehat{\mathbf{L}}_{\acute{y}a}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} \\ & +2\epsilon^{ab}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{L}}_{\acute{w}c}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\alpha}\, \widehat{\mathbf{L}}_{\acute{y}b}\, \widehat{\mathbf{Q}}^{c\alpha}_{\acute{z}} - 2\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\alpha}\, \widehat{\mathbf{L}}_{\acute{w}b}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\, \widehat{\mathbf{L}}_{\acute{y}a}\, \widehat{\mathbf{Q}}^{b\beta}_{\acute{z}} \Big] \\ & \times \sum_{N=2}^{7} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{u}\acute{v}} V_{d_{rN}}\right\} \left\{\sum_{s=1}^{2} \mathcal{B}^{(s)}_{\acute{w}\acute{x},\acute{y}\acute{z}} U_{d_{sN}}\right\}}{m_{d_N}} \\ & -16 \Big[\widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{w}\alpha}\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{x}} + \epsilon_{ab}\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\alpha}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{w}\beta}\, \widehat{\mathbf{Q}}^{b\beta}_{\acute{x}} \Big] \\ & \times \sum_{N=2}^{7} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{u}\acute{v}} V_{d_{rN}}\right\} \left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{w}\acute{x}} U_{d_{sN}}\right\}}{m_{d_N}}. \end{aligned} \tag{33}$$

(ii) **Evaluating $\widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}\alpha}\ \widehat{\mathsf{H}}^{(10_r)\alpha}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}}$**
Here we have couplings involving the 10-plet of Higgs triplet fields $\widehat{\mathsf{H}}^{(10_r)\alpha}$ $(r = 1, 2)$. The Higgs triplet fields are all superheavy and their elimination gives rise to $\mathsf{B} - \mathsf{L} = -2$ operators involving six fields all of which are matter fields. One also gets B and L violating but $\mathsf{B} - \mathsf{L}$ preserving operators with four fields. Thus together we get the following set of operators

$$\begin{aligned} W_{\mathcal{B}}^{(Triplet)} = & \imath 4\sqrt{2} \Big[\epsilon^{\beta\gamma\rho}\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\beta}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\gamma}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\alpha}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\rho} + \epsilon^{\alpha\beta\gamma}\epsilon^{\rho\sigma\lambda}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{u}\alpha}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\rho}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\sigma}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\gamma}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\lambda} \\ & +\epsilon^{bc}\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{L}}_{\acute{w}b}\, \widehat{\mathbf{L}}_{\acute{x}c}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\alpha}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} + \epsilon^{ab}\epsilon^{\alpha\beta\gamma}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{u}\alpha}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\, \widehat{\mathbf{L}}_{\acute{w}a}\, \widehat{\mathbf{L}}_{\acute{x}b}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\gamma}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} \\ & +2\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{L}}_{\acute{w}b}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\alpha}\, \widehat{\mathbf{Q}}^{b\beta}_{\acute{z}} + 2\epsilon^{\alpha\beta\gamma}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{u}\alpha}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\, \widehat{\mathbf{L}}_{\acute{w}a}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\rho}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\gamma}\, \widehat{\mathbf{Q}}^{a\rho}_{\acute{z}} \Big] \\ & \times \sum_{N=1}^{8} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{u}\acute{v}} V_{t_{rN}}\right\} \left\{\sum_{s=1}^{2} B^{(s)}_{\acute{w}\acute{x},\acute{y}\acute{z}} U_{t_{sN}}\right\}}{m_{t_N}} \\ & +8 \Big[2\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{w}}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{x}\alpha} + 2\epsilon^{\alpha\beta\gamma}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{u}\alpha}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\, \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{w}}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{x}\gamma} \\ & - \epsilon_{\alpha\beta\gamma}\epsilon_{bc}\, \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\, \widehat{\mathbf{L}}_{\acute{v}a}\, \widehat{\mathbf{Q}}^{b\beta}_{\acute{w}}\, \widehat{\mathbf{Q}}^{c\gamma}_{\acute{x}} + 2\epsilon_{ab}\, \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{u}\alpha}\, \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\, \widehat{\mathbf{Q}}^{a\beta}_{\acute{w}}\, \widehat{\mathbf{Q}}^{b\alpha}_{\acute{x}} \Big] \\ & \times \sum_{N=1}^{8} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{u}\acute{v}} V_{t_{rN}}\right\} \left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{w}\acute{x}} U_{t_{sN}}\right\}}{m_{t_N}}. \end{aligned} \tag{34}$$

In the above the top three lines give $\mathsf{B} - \mathsf{L} = -2$ operators which involve six matter fields. The bottom two lines give $\mathsf{B} - \mathsf{L} = 0$ operators which involve four matter fields of which the operators $\widehat{\mathbf{U}}^{\mathsf{c}}\widehat{\mathbf{D}}^{\mathsf{c}}\widehat{\mathbf{E}}^{\mathsf{c}}\widehat{\mathbf{U}}^{\mathsf{c}}$ and $\widehat{\mathbf{Q}}\widehat{\mathbf{L}}\widehat{\mathbf{Q}}\widehat{\mathbf{Q}}$ violate B and L.

5.3. Operators arising from $\widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(120)k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}}$

$$W_{\mathcal{C}} = \mathcal{C}_{\acute{w}\acute{x},\acute{y}\acute{z}}\ \widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(120)k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}}, \tag{35}$$

(i) **Evaluating $\widehat{\mathbf{M}}_{\acute{w}i}\ \widehat{\mathbf{M}}_{\acute{x}j}\ \widehat{\mathbf{M}}_{\acute{y}a}\ \widehat{\mathsf{H}}^{(120)a}\ \widehat{\mathbf{M}}^{ij}_{\acute{z}}$**
As in the case of $\widehat{\mathsf{H}}^{(10)a}$ we expand $\widehat{\mathsf{H}}^{(120)a}$ in terms of light and heavy Higgs doublet modes. This leads to $\mathsf{B}-\mathsf{L}=-2$ operators with four matter fields and a light Higgs doublet field and operators with six matter fields as follows

$$\begin{aligned} W_{\mathcal{C}}^{(Doublet)} = \; & U_{d_{31}}\mathcal{C}_{\acute{w}\acute{x},\acute{y}\acute{z}}\Big[\epsilon^{\alpha\beta\gamma}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\ \widehat{\mathbf{L}}_{\acute{y}a}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\gamma}\ \widehat{\mathbf{H}}^{a}_{\mathbf{u}} + \epsilon^{ab}\ \widehat{\mathbf{L}}_{\acute{w}a}\ \widehat{\mathbf{L}}_{\acute{x}b}\ \widehat{\mathbf{L}}_{\acute{y}c}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}}\ \widehat{\mathbf{H}}^{c}_{\mathbf{u}} \\ & +2\ \widehat{\mathbf{L}}_{\acute{w}a}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\alpha}\ \widehat{\mathbf{L}}_{\acute{y}b}\ \widehat{\mathbf{Q}}^{a\alpha}_{\acute{z}}\ \widehat{\mathbf{H}}^{b}_{\mathbf{u}}\Big] \\ & +\frac{\imath 4}{\sqrt{3}}f^{(120-)}_{\acute{u}\acute{v}}\mathcal{C}_{\acute{w}\acute{x},\acute{y}\acute{z}}\Big[\epsilon^{\alpha\beta\gamma}\epsilon^{ab}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{u}}\ \widehat{\mathbf{L}}_{\acute{v}a}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\ \widehat{\mathbf{L}}_{\acute{y}b}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\gamma} \\ & -\epsilon^{\beta\gamma\rho}\ \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\beta}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\gamma}\ \widehat{\mathbf{L}}_{\acute{y}a}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\rho} + \epsilon^{ab}\epsilon^{cd}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{u}}\ \widehat{\mathbf{L}}_{\acute{v}a}\ \widehat{\mathbf{L}}_{\acute{w}c}\ \widehat{\mathbf{L}}_{\acute{x}d}\ \widehat{\mathbf{L}}_{\acute{y}b}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} \\ & -\epsilon^{bc}\ \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\alpha}\ \widehat{\mathbf{L}}_{\acute{w}b}\ \widehat{\mathbf{L}}_{\acute{x}c}\ \widehat{\mathbf{L}}_{\acute{y}a}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} + 2\epsilon^{ab}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{u}}\ \widehat{\mathbf{L}}_{\acute{v}a}\ \widehat{\mathbf{L}}_{\acute{w}c}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\alpha}\ \widehat{\mathbf{L}}_{\acute{y}b}\ \widehat{\mathbf{Q}}^{c\alpha}_{\acute{z}} \\ & -2\ \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\alpha}\ \widehat{\mathbf{L}}_{\acute{w}b}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\ \widehat{\mathbf{L}}_{\acute{y}a}\ \widehat{\mathbf{Q}}^{b\beta}_{\acute{z}}\Big]\sum_{N=2}^{7}\frac{V_{d_{3N}}U_{d_{3N}}}{m_{d_N}}. \end{aligned} \tag{36}$$

(ii) **Evaluating $\widehat{\mathbf{M}}_{\acute{w}i}\ \widehat{\mathbf{M}}_{\acute{x}j}\ \widehat{\mathbf{M}}_{\acute{y}\alpha}\ \widehat{\mathsf{H}}^{(120)\alpha}\ \widehat{\mathbf{M}}^{ij}_{\acute{z}}$**
Here all the Higgs triplet fields $\widehat{\mathsf{H}}^{(120)\alpha}$ are superheavy and their elimination leads to $\mathsf{B}-\mathsf{L}=-2$ operators with six matter fields as follows

$$\begin{aligned} W_{\mathcal{C}}^{(Triplet)} = \; & \frac{\imath 4}{\sqrt{3}}f^{(120-)}_{\acute{u}\acute{v}}\mathcal{C}_{\acute{w}\acute{x},\acute{y}\acute{z}}\Big[\epsilon^{\beta\gamma\rho}\ \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\ \widehat{\mathbf{L}}_{\acute{v}a}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\beta}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\gamma}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\alpha}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\rho} \\ & +\epsilon^{\alpha\beta\gamma}\epsilon^{\rho\sigma\lambda}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{u}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{w}\rho}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\sigma}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\gamma}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{z}\lambda} + \epsilon^{bc}\ \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\ \widehat{\mathbf{L}}_{\acute{v}a}\ \widehat{\mathbf{L}}_{\acute{w}b}\ \widehat{\mathbf{L}}_{\acute{x}c}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\alpha}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} \\ & +\epsilon^{ab}\epsilon^{\alpha\beta\gamma}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{u}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\ \widehat{\mathbf{L}}_{\acute{w}a}\ \widehat{\mathbf{L}}_{\acute{x}b}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\gamma}\ \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{z}} + 2\ \widehat{\mathbf{Q}}^{a\alpha}_{\acute{u}}\ \widehat{\mathbf{L}}_{\acute{v}a}\ \widehat{\mathbf{L}}_{\acute{w}b}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\beta}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\alpha}\ \widehat{\mathbf{Q}}^{b\beta}_{\acute{z}} \\ & +2\epsilon^{\alpha\beta\gamma}\ \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{u}\alpha}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{v}\beta}\ \widehat{\mathbf{L}}_{\acute{w}a}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\rho}\ \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{y}\gamma}\ \widehat{\mathbf{Q}}^{a\rho}_{\acute{z}}\Big]\sum_{N=1}^{8}\frac{V_{t_{3N}}U_{t_{3N}}}{m_{t_N}}. \end{aligned} \tag{37}$$

5.4. Operators arising from $\widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(\overline{126})k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}}$

$$W_{\mathcal{D}} = \mathcal{D}_{\acute{w}\acute{x},\acute{y}\acute{z}}\ \widehat{\mathsf{M}}_{\acute{w}i}\ \widehat{\mathsf{M}}_{\acute{x}j}\ \widehat{\mathsf{M}}_{\acute{y}k}\ \widehat{\mathsf{H}}^{(\overline{126})k}\ \widehat{\mathsf{M}}^{ij}_{\acute{z}}, \tag{38}$$

This term does not generate any $\mathsf{B}-\mathsf{L}=-2$ operators involving only the SM fields because, firstly $U_{d_{41}}=0$ and secondly because there is no $\overline{5}$ of $\mathsf{SU}(5)$ in $\overline{126}$ and thus a mass term involving 5 and $\overline{5}$ cannot be written.

6. Conclusion

As is well known there are a variety of reasons why the gauge group SO(10) as the unifying group of electro-weak and strong interactions is preferred over the group SU(5). Thus it is of relevance to ask what phenomena might set SO(10) apart from SU(5). One clear distinction between the two is the existence of B − L violating interactions in SO(10) while SU(5) couplings are B−L preserving. Thus the observation of B−L violating processes would act as discriminants between SU(5) and SO(10). The analysis of this work is to compute the B − L interactions in the framework of a class of SO(10) models with a natural doublet-triplet splitting using the missing partner mechanism. The missing partner mechanism involves two Higgs sectors, one heavy and one light. If there is an excess of the number of Higgs doublet pairs in the light Higgs sector relative to the number of Higgs doublet pairs in the heavy Higgs sector by one, then one pair of

Higgs doublets will remain light when the light and heavy sectors mix. In the analysis of [28] a classification of $\mathsf{SO}(10)$ models with a missing mechanism for the generation of a light Higgs double pair was given. In this analysis we considered one specific model where the Higgs content consists of a heavy sector with $126 + \overline{126} + 210$ of Higgs fields and a light sector consisting of $2 \times 10 + 120$ of Higgs fields. From the heavy sector one has 3 doublets of Higgs pairs and in the light sector one has 4 pairs of Higgs pairs. When the light and heavy sector mix, we are left with just one light pair of Higgs doublets. Further all of the Higgs triplets/anti-triplets from the light sector and from the heavy sectors become heavy after the light and the heavy Higgs sectors mix.

The symmetry breaking in the system proceeds in the following way. The $\mathsf{SU}(5)$ singlets in $126+\overline{126}$ and in 210 develop a VEV via spontaneous breaking. The breaking produces $10+\overline{10}$ of Goldstone bosons which correspond to the breaking of $\mathsf{SO}(10)$ to $\mathsf{SU}(5) \times \mathsf{U}(1)$. Thus there are three pairs of $10 + \overline{10}$ of fields which arise from 120, $126 + \overline{126}$ and from 210. One combination of these form the $10 + \overline{10}$ Goldstone bosons which are absorbed by the $10 + \overline{10}$ of vector bosons in 45 of $\mathsf{SO}(10)$ to become superheavy. The other two pairs of $10 + \overline{10}$ are massive and their elimination leads to $\mathsf{B} - \mathsf{L}$ violating interactions computed in this work. This complements the analysis done in a previous work where the $\mathsf{B} - \mathsf{L}$ violating interactions were computed by integration over the $5 + \bar{5}$ and $45 + \overline{45}$ of heavy Higgs Higgs fields. The analysis leads to five field operators and six fields operators in the superpotential which result in dimension 7 and dimension 9 operators in the Lagrangian after dressings. One operator of specific interest is $\widehat{\mathbf{L}}\widehat{\mathbf{L}}\widehat{\mathbf{E}}^{\mathsf{c}}\widehat{\mathbf{D}}^{\mathsf{c}}\widehat{\mathbf{D}}^{\mathsf{c}}\widehat{\mathbf{U}}^{\mathsf{c}}$ which produces a purely leptonic decay of the neutron, i.e., $n \to \nu l_i l_j$. The analysis presented here could be utilized for analyses of $n - \bar{n}$ oscillations and for investigations of GUT scale baryogengesis in the early universe.

Appendix A: Notation and particle Content

We begin by displaying the decomposition of 16−plet of matter and 10−, 120−, $\overline{126}$− and 210−plets of Higgs of $\mathsf{SO}(10)$ in terms of $\mathsf{SU}(5)$ representations. Thus we have

$$\begin{aligned}
16 &= 1(-5)\left[\widehat{\mathsf{M}}_{\acute{x}}\right] + \bar{5}(3)\left[\widehat{\mathsf{M}}_{\acute{x}i}\right] + 10(-1)\left[\widehat{\mathsf{M}}^{ij}_{\acute{x}}\right],\\
10_{\mathsf{r}} &= 5(2)\left[\widehat{\mathsf{H}}^{(10_r)i}\right] + \bar{5}(-2)\left[\widehat{\mathsf{H}}^{(10_r)}_i\right],\\
120\ (\Sigma) &= 5(2)\left[\widehat{\mathsf{H}}^{(120)i}\right] + \bar{5}(-2)\left[\widehat{\mathsf{H}}^{(120)}_i\right] + 10(-6)\left[\widehat{\mathsf{H}}^{(120)ij}\right] + \overline{10}(6)\left[\widehat{\mathsf{H}}^{(120)}_{ij}\right] + 45(2)\left[\widehat{\mathsf{H}}^{(120)ij}_k\right]\\
&\quad +\overline{45}(-2)\left[\widehat{\mathsf{H}}^{(120)k}_{ij}\right],\\
\overline{126}\ (\overline{\Delta}) &= 1(10)\left[\widehat{\mathsf{H}}^{(\overline{126})}\right] + 5(2)\left[\widehat{\mathsf{H}}^{(\overline{126})i}\right] + \overline{10}(6)\left[\widehat{\mathsf{H}}^{(\overline{126})}_{ij}\right] + 15(-6)\left[\widehat{\mathsf{H}}^{(\overline{126})ij}_{(S)}\right] + \overline{45}(-2)\left[\widehat{\mathsf{H}}^{(\overline{126})k}_{ij}\right]\\
&\quad +50(2)\left[\widehat{\mathsf{H}}^{(\overline{126})ijk}_{lm}\right],\\
210\ (\Phi) &= 1(0)\left[\widehat{\mathsf{H}}^{(210)}\right] + 5(-8)\left[\widehat{\mathsf{H}}^{(210)i}\right] + \bar{5}(8)\left[\widehat{\mathsf{H}}^{(210)}_i\right] + 10(4)\left[\widehat{\mathsf{H}}^{(210)ij}\right] + \overline{10}(-4)\left[\widehat{\mathsf{H}}^{(210)}_{ij}\right]\\
&\quad +24(0)\left[\widehat{\mathsf{H}}^{(120)i}_j\right] + 40(-4)\left[\widehat{\mathsf{H}}^{(210)ijk}_l\right] + \overline{40}(4)\left[\widehat{\mathsf{H}}^{(210)l}_{ijk}\right] + 75(0)\left[\widehat{\mathsf{H}}^{(210)ij}_{kl}\right],
\end{aligned} \tag{1}$$

where $i,\ j = 1, ..., 5$ are $\mathsf{SU}(5)$ indices, $\acute{u},\ \acute{v},\ \acute{w},\ \acute{x},\ \acute{y},\ \acute{z} = 1,\ 2,\ 3$ represent generation indices and $r,\ s = 1,\ 2$ count the number of 10 plets of $\mathsf{SO}(10)$ used in our model of the missing partner mechanism. Latin letters with the symbol $\widehat{}$ represent superfields. The $\mathsf{SU}(5)$ matter superfields contain the following MSSM chiral superfields

$$\begin{aligned}
\widehat{\mathsf{M}}_{\acute{x}} = \widehat{\boldsymbol{\nu}}^{\mathsf{c}}_{\acute{x}}; \qquad \widehat{\mathsf{M}}_{\acute{x}\alpha} = \widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\alpha}; \qquad \widehat{\mathsf{M}}_{\acute{x}a} = \widehat{\mathbf{L}}_{\acute{x}a};\\
\widehat{\mathsf{M}}^{a\alpha}_{\acute{x}} = \widehat{\mathbf{Q}}^{a\alpha}_{\acute{x}}; \qquad \widehat{\mathsf{M}}^{\alpha\beta}_{\acute{x}} = \epsilon^{\alpha\beta\gamma}\widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{x}\gamma}; \qquad \widehat{\mathsf{M}}^{ab}_{\acute{x}} = \epsilon^{ab}\widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{x}},
\end{aligned} \tag{2}$$

where $\alpha,\ \beta,\ \gamma = 1,\ 2,\ 3$ are $\mathsf{SU}(3)$ color indices, while $a,\ b = 4,\ 5$ are $\mathsf{SU}(2)$ weak indices and the superscript $^{\mathsf{c}}$ denotes charge conjugation. Scalar (denoted by $\tilde{}$) and two-component fermionic fields residing in the MSSM superfields are identified to be

$$\widehat{\mathbf{D}}^{\mathsf{c}}_{\acute{x}\alpha} \supseteq \left(\widetilde{\mathbf{d}}^{*}_{R\acute{x}\alpha}\ ,\ \mathbf{d}^{\mathsf{c}}_{L\acute{x}\alpha}\right), \qquad \widehat{\mathbf{U}}^{\mathsf{c}}_{\acute{x}\alpha} \supseteq (\widetilde{\mathbf{u}}^{*}_{R\acute{x}\alpha}\ ,\ \mathbf{u}^{\mathsf{c}}_{L\acute{x}\alpha}), \qquad \widehat{\mathbf{E}}^{\mathsf{c}}_{\acute{x}} \supseteq (\widetilde{\mathbf{e}}^{*}_{R\acute{x}}\ ,\ \mathbf{e}^{\mathsf{c}}_{L\acute{x}}), \qquad \widehat{\boldsymbol{\nu}}^{\mathsf{c}}_{\acute{x}} \supseteq (\widetilde{\boldsymbol{\nu}}^{*}_{R\acute{x}}\ ,\ \boldsymbol{\nu}^{\mathsf{c}}_{L\acute{x}})$$

$$\widehat{\mathbf{L}}_{\acute{x}a} = \begin{pmatrix}\widehat{\boldsymbol{\nu}}_{\acute{x}}\\ \widehat{\mathbf{E}}_{\acute{x}}\end{pmatrix} \supseteq \left(\widetilde{\mathbf{L}}_{\acute{x}a}\ ,\ \mathbf{L}_{\acute{x}a}\right), \qquad \widehat{\mathbf{Q}}^{a\alpha}_{\acute{x}} = \begin{pmatrix}\widehat{\mathbf{U}}^{\alpha}_{\acute{x}}\\ \widehat{\mathbf{D}}^{\alpha}_{\acute{x}}\end{pmatrix} \supseteq \left(\widetilde{\mathbf{Q}}^{a\alpha}_{\acute{x}}\ ,\ \mathbf{Q}^{a\alpha}_{\acute{x}}\right), \qquad \widehat{\mathbf{H}}_{\mathbf{u}} \supseteq \left(\mathbf{H}_{\mathbf{u}}\ ,\ \widetilde{\mathbf{H}}_{\mathbf{u}}\right)$$

where

$$\widetilde{\mathbf{L}}_{\acute{x}a} = \begin{pmatrix}\widetilde{\boldsymbol{\nu}}_{L\acute{x}}\\ \widetilde{\mathbf{e}}_{L\acute{x}}\end{pmatrix}, \qquad \mathbf{L}_{\acute{x}a} = \begin{pmatrix}\boldsymbol{\nu}_{L\acute{x}}\\ \mathbf{e}_{L\acute{x}}\end{pmatrix}, \qquad \widetilde{\mathbf{Q}}^{a\alpha}_{\acute{x}} = \begin{pmatrix}\widetilde{\mathbf{u}}^{\alpha}_{L\acute{x}}\\ \widetilde{\mathbf{d}}^{\alpha}_{L\acute{x}}\end{pmatrix}, \qquad \mathbf{Q}_{\acute{x}} = \begin{pmatrix}\mathbf{u}^{\alpha}_{L\acute{x}}\\ \mathbf{d}^{\alpha}_{L\acute{x}}\end{pmatrix}$$

$$\mathbf{H}_{\mathbf{u}}{}^{a} = \begin{pmatrix}\mathbf{H}^{+}_{\mathbf{u}}\\ \mathbf{H}^{\mathbf{0}}_{\mathbf{u}}\end{pmatrix}, \qquad \widetilde{\mathbf{H}}^{a}_{\mathbf{u}} = \begin{pmatrix}\widetilde{\mathbf{H}}^{+}_{\mathbf{u}}\\ \widetilde{\mathbf{H}}^{\mathbf{0}}_{\mathbf{u}}\end{pmatrix}$$

Appendix B: The light Higgs doublet pair of the Doublet Mass Matrix

From our previous paper [30], the doublet mass matrix M_d is diagonalized by two unitary matrices U_d and V_d:

$$U_d{}^{\dagger} M_d \mathsf{V}_{\mathsf{d}} = \mathrm{diag}\,(0, m_{d_2}, m_{d_3}, \cdots, m_{d_7})\,. \tag{3}$$

The mass eigenstate Higgs doublet fields are expressed in terms of the original Higgs doublet fields through

$$\begin{pmatrix} {}^{(\overline{5}_{10_1})}\widehat{\mathbf{D}}'_a\\ {}^{(\overline{5}_{10_2})}\widehat{\mathbf{D}}'_a\\ {}^{(\overline{5}_{120})}\widehat{\mathbf{D}}'_a\\ {}^{(\overline{5}_{126})}\widehat{\mathbf{D}}'_a\\ {}^{(\overline{5}_{210})}\widehat{\mathbf{D}}'_a\\ {}^{(\overline{45}_{120})}\widehat{\mathbf{D}}'_a\\ {}^{(\overline{45}_{\overline{126}})}\widehat{\mathbf{D}}'_a \end{pmatrix} = V_d{}^{\dagger} \begin{pmatrix} {}^{(\overline{5}_{10_1})}\widehat{\mathbf{D}}_a\\ {}^{(\overline{5}_{10_2})}\widehat{\mathbf{D}}_a\\ {}^{(\overline{5}_{120})}\widehat{\mathbf{D}}_a\\ {}^{(\overline{5}_{126})}\widehat{\mathbf{D}}_a\\ {}^{(\overline{5}_{210})}\widehat{\mathbf{D}}_a\\ {}^{(\overline{45}_{120})}\widehat{\mathbf{D}}_a\\ {}^{(\overline{45}_{\overline{126}})}\widehat{\mathbf{D}}_a \end{pmatrix}; \quad \begin{pmatrix} {}^{(5_{10_1})}\widehat{\mathbf{D}}'^a\\ {}^{(5_{10_2})}\widehat{\mathbf{D}}'^a\\ {}^{(5_{120})}\widehat{\mathbf{D}}'^a\\ {}^{(5_{\overline{126}})}\widehat{\mathbf{D}}'^a\\ {}^{(5_{210})}\widehat{\mathbf{D}}'^a\\ {}^{(45_{120})}\widehat{\mathbf{D}}'^a\\ {}^{(45_{126})}\widehat{\mathbf{D}}'^a \end{pmatrix} = U_d{}^{\dagger} \begin{pmatrix} {}^{(5_{10_1})}\widehat{\mathbf{D}}^a\\ {}^{(5_{10_2})}\widehat{\mathbf{D}}^a\\ {}^{(5_{120})}\widehat{\mathbf{D}}^a\\ {}^{(5_{\overline{126}})}\widehat{\mathbf{D}}^a\\ {}^{(5_{210})}\widehat{\mathbf{D}}^a\\ {}^{(45_{120})}\widehat{\mathbf{D}}^a\\ {}^{(45_{126})}\widehat{\mathbf{D}}^a \end{pmatrix}. \tag{4}$$

The light Higgs Doublet pair is $\left({}^{(\overline{5}_{10_1})}\widehat{\mathbf{D}}'_a, {}^{(5_{10_1})}\widehat{\mathbf{D}}'^a\right) \equiv (\widehat{\mathbf{H}}_{\mathbf{d}a}, \widehat{\mathbf{H}}^a_{\mathbf{u}})$. Thus, the inverse transformation of Eq.(4) gives

$$\begin{aligned} &{}^{(\overline{5}_{10_1})}\widehat{\mathbf{D}}_a = V_{d_{11}}\widehat{\mathbf{H}}_{\mathbf{d}a} + \cdots, \quad {}^{(\overline{5}_{10_2})}\widehat{\mathbf{D}}_a = V_{d_{21}}\widehat{\mathbf{H}}_{\mathbf{d}a} + \cdots, \quad {}^{(\overline{5}_{120})}\widehat{\mathbf{D}}_a = V_{d_{31}}\widehat{\mathbf{H}}_{\mathbf{d}a} + \cdots,\\ &{}^{(\overline{5}_{126})}\widehat{\mathbf{D}}_a = V_{d_{41}}\widehat{\mathbf{H}}_{\mathbf{d}a} + \cdots, \quad {}^{(\overline{5}_{210})}\widehat{\mathbf{D}}_a = V_{d_{51}}\widehat{\mathbf{H}}_{\mathbf{d}a} + \cdots, \quad {}^{(\overline{45}_{120})}\widehat{\mathbf{D}}_a = V_{d_{61}}\widehat{\mathbf{H}}_{\mathbf{d}a} + \cdots,\\ &{}^{(\overline{45}_{\overline{126}})}\widehat{\mathbf{D}}_a = V_{d_{71}}\widehat{\mathbf{H}}_{\mathbf{d}a} + \cdots, \end{aligned} \tag{5}$$

and

$$\begin{aligned} &{}^{(5_{10_1})}\widehat{\mathbf{D}}^a = U_{d_{11}}\widehat{\mathbf{H}}^a_{\mathbf{u}} + \cdots, \quad {}^{(5_{10_2})}\widehat{\mathbf{D}}^a = U_{d_{21}}\widehat{\mathbf{H}}^a_{\mathbf{u}} + \cdots, \quad {}^{(5_{120})}\widehat{\mathbf{D}}^a = U_{d_{31}}\widehat{\mathbf{H}}^a_{\mathbf{u}} + \cdots,\\ &{}^{(5_{126})}\widehat{\mathbf{D}}^a = U_{d_{41}}\widehat{\mathbf{H}}^a_{\mathbf{u}} + \cdots, \quad {}^{(5_{210})}\widehat{\mathbf{D}}^a = U_{d_{51}}\widehat{\mathbf{H}}^a_{\mathbf{u}} + \cdots, \quad {}^{(45_{120})}\widehat{\mathbf{D}}^a = U_{d_{61}}\widehat{\mathbf{H}}^a_{\mathbf{u}} + \cdots,\\ &{}^{(45_{126})}\widehat{\mathbf{D}}^a = U_{d_{71}}\widehat{\mathbf{H}}^a_{\mathbf{u}} + \cdots. \end{aligned} \tag{6}$$

Vanishing elements of the matrices U_d and V_d are $V_{d_{41}},\ V_{d_{51}},\ V_{d_{71}}$ and $V_{d_{41}},\ U_{d_{51}},\ V_{d_{71}}$. Hence the couplings involving $f^{(\overline{126}+)}_{\acute{x}\acute{y}}$ vanishes. Therefore, the quark and charged lepton masses arise only from the Yukawa couplings $f^{(10_1+)}_{\acute{x}\acute{y}}$, $f^{(10_2+)}_{\acute{x}\acute{y}}$ and $f^{(120-)}_{\acute{x}\acute{y}}$.

References

[1] S. Weinberg, 1979 *Phys. Rev. Lett.* **43**, 1566
[2] F. Wilczek and A. Zee, 1979 *Phys. Rev. Lett.* **43**, 1571
[3] S. Weinberg, 1980 *Phys. Rev. D* **22**, 1694
[4] H. A. Weldon and A. Zee, 1980 *Nucl. Phys. B* **173**, 269
[5] R. N. Mohapatra and R. E. Marshak, 1980 *Phys. Rev. Lett.* **44**, 1316 Erratum: [1980 Phys. Rev. Lett. **44**, 1643]
[6] L. N. Chang and N. P. Chang, 1980 *Phys. Rev. Lett.* **45**, 1540
[7] T. K. Kuo and S. T. Love, 1980 *Phys. Rev. Lett.* **45**, 93
[8] J. F. Nieves, 1981 *Nucl. Phys. B* **189**, 182
[9] S. Rao and R. Shrock, 1982 *Phys. Lett.* **116B**, 238
[10] W. E. Caswell, J. Milutinovic and G. Senjanovic, 1983 *Phys. Lett.* **122B**, 373
[11] S. Rao and R. E. Shrock, 1984 *Nucl. Phys. B* **232**, 143
[12] C. N. Leung, S. T. Love and S. Rao, 1986 *Z. Phys. C* **31**, 433
[13] W. Buchmuller and D. Wyler, 1986 *Nucl. Phys. B* **268**, 621
[14] K. S. Babu and C. N. Leung, 2001 *Nucl. Phys. B* **619**, 667 (*Preprint* hep-ph/0106054)
[15] F. Bonnet, D. Hernandez, T. Ota and W. Winter, 2009 *JHEP* **0910**, 076 (*Preprint* arXiv:0907.3143 [hep-ph])
[16] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, 2010 *JHEP* **1010**, 085 (*Preprint* arXiv:1008.4884 [hep-ph])
[17] F. del Aguila, A. Aparici, S. Bhattacharya, A. Santamaria and J. Wudka, 2012 *JHEP* **1206**, 146 (*Preprint* arXiv:1204.5986 [hep-ph])
[18] P. W. Angel, N. L. Rodd and R. R. Volkas, 2013 *Phys. Rev. D* **87**, no. 7, 073007 (*Preprint* arXiv:1212.6111 [hep-ph]).
[19] M. B. Krauss, D. Meloni, W. Porod and W. Winter, 2013 *JHEP* **1305**, 121 (*Preprint* arXiv:1301.4221 [hep-ph])
[20] C. Degrande, 2014 *JHEP* **1402**, 101 (*Preprint* arXiv:1308.6323 [hep-ph])
[21] G. Chalons and F. Domingo, 2014 *Phys. Rev. D* **89**, no. 3, 034004 (*Preprint* arXiv:1303.6515 [hep-ph])
[22] L. Lehman, 2014 *Phys. Rev. D* **90**, no. 12, 125023 (*Preprint* arXiv:1410.4193 [hep-ph])
[23] S. Enomoto and N. Maekawa, 2011 *Phys. Rev. D* **84**, 096007 (*Preprint* arXiv:1107.3713 [hep-ph])
[24] K. S. Babu and R. N. Mohapatra, 2012 *Phys. Rev. Lett.* **109**, 091803 (*Preprint* arXiv:1207.5771 [hep-ph])
[25] K. S. Babu and R. N. Mohapatra, 2012 *Phys. Rev. D* **86**, 035018 (*Preprint* arXiv:1203.5544 [hep-ph])
[26] A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, 1982 *Phys. Lett.* **115B**, 380
[27] K. S. Babu, I. Gogoladze and Z. Tavartkiladze, 2007 *Phys. Lett. B* **650**, 49 (*Preprint* hep-ph/0612315)
[28] K. S. Babu, I. Gogoladze, P. Nath and R. M. Syed, 2012 *Phys. Rev. D* **85**, 075002 (*Preprint* arXiv:1112.5387 [hep-ph])
[29] L. Du, X. Li and D. X. Zhang, 2014 *JHEP* **1404**, 027 (*Preprint* arXiv:1312.1786 [hep-ph])
[30] P. Nath and R. M. Syed, 2016 *Phys. Rev. D* **93**, no. 5, 055005 (*Preprint* arXiv:1508.00585 [hep-ph])
[31] T.E. Clark, T.K. Kuo, and N. Nakagawa, 1982 Phys. lett. B 115, 26; C.S. Aulakh and R.N. Mohapatra, 1983 *Phys. Rev. D* 28, 217 ().
[32] C.S. Aulakh, B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, 2004 *Phys. Lett. B* 588, 196
[33] C.S. Aulakh and S.K. Garg, 2006 *Nucl. Phys. B* 757, 47 () (*Preprint* arXiv:hep-ph/0512224)
[34] C. S. Aulakh and S. K. Garg, 2012 *Nucl. Phys. B* **857**, 101; C.S. Aulakh and A. Girdhar, 2005 Nucl. Phys. B 711, 275; (*Preprint* arXiv:hep-ph/0405074); B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, 2004 *Phys. Rev. D* 70, 035007. (*Preprint* arXiv:hep- ph/0402122)
[35] R. N. Mohapatra and B. Sakita, 1980 *Phys. Rev. D* **21**, 1062
[36] P. Nath and R. M. Syed, 2001 *Phys. Lett. B* **506**, 68 Erratum: [2001 Phys. Lett. B **508**, 216] (*Preprint* hep-ph/0103165)
[37] P. Nath and R. M. Syed, 2001 *Nucl. Phys. B* **618**, 138 (*Preprint* hep-th/0109116)
[38] P. Nath and R. M. Syed, 2004 *Nucl. Phys. B* **676**, 64 (*Preprint* hep-th/0310178)
[39] R. M. Syed, *Preprint* hep-ph/0508153
[40] K. S. Babu, I. Gogoladze, P. Nath and R. M. Syed, 2005 *Phys. Rev. D* **72**, 095011 (*Preprint* hep-ph/0506312)
[41] K. S. Babu, I. Gogoladze, P. Nath and R. M. Syed, 2006 *Phys. Rev. D* **74**, 075004 (*Preprint* hep-ph/0607244)
[42] C. S. Aulakh and A. Girdhar, 2005 *Int. J. Mod. Phys. A* **20**, 865 (*Preprint* hep-ph/0204097)
[43] Z. Y. Chen, D. X. Zhang and X. Z. Bai, 2017 *Int. J. Mod. Phys. A* **32**, no. 36, 1750207 (*Preprint* arXiv:1707.00580 [hep-ph])
[44] A. H. Chamseddine, R. L. Arnowitt and P. Nath, 1982 *Phys. Rev. Lett.* **49**, 970
[45] P. Nath, A. H. Chamseddine and R. L. Arnowitt, 1985 *Phys. Rev. D* **32**, 2348
[46] P. Nath and P. Fileviez Perez, 2007 *Phys. Rept.* **441**, 191 (*Preprint* hep-ph/0601023)